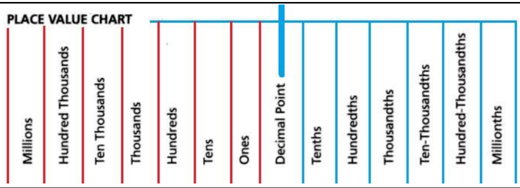
Name:……………….………… Set:………….

Maths
Knowledge Organiser

Higher

- This booklet includes references to each unit you will cover in your learning journey.
- It is to be used as both a reference and revision tool.
- Keep it with you in your planner wallet so that it is available to you in lessons.
- Your weekly ILTs will also reference different units and require you to complete a specific revision task.

1. Fractions and Decimals

Topic/Skill	Definition/Tips	Example
1. Integer	A **whole number** that can be positive, negative or zero.	$-3, 0, 92$
2. Decimal	A number with a **decimal point** in it. Can be positive or negative.	$3.7, 0.94, -24.07$
3. Negative Number	A number that is **less than zero**. Can be decimals.	$-8, -2.5$
4. Addition	To find the **total**, or **sum**, of two or more numbers. 'add', 'plus', 'sum'	$3 + 2 + 7 = 12$
5. Subtraction	To find the **difference** between two numbers. To find out how many are left when some are taken away. 'minus', 'take away', 'subtract'	$10 - 3 = 7$
6. Multiplication	Can be thought of as **repeated addition**. 'multiply', 'times', 'product'	$3 \times 6 = 6 + 6 + 6 = 18$
7. Division	Splitting into equal parts or groups. The process of calculating the **number of times one number is contained within another one**. 'divide', 'share'	$20 \div 4 = 5$ $\frac{20}{4} = 5$
8. Remainder	The amount '**left over**' after dividing one integer by another.	The remainder of $20 \div 6$ is 2, because 6 divides into 20 exactly 3 times, with 2 left over.
9. Place Value	The **value** of where a **digit** is within a number.	In 726, the value of the 2 is 20, as it is in the 'tens' column.
10. Place Value Columns	The names of the columns that **determine the value of each digit**. The 'ones' column is also known as the 'units' column.	
11. Rounding	To make a number simpler but keep its value close to what it was. If the **digit to the right** of the rounding digit is **less than 5, round down**. If the **digit to the right** of the rounding digit is **5 or more, round up**.	74 rounded to the nearest ten is 70, because 74 is closer to 70 than 80. 152,879 rounded to the nearest thousand is 153,000.
12. Decimal Place	The **position** of a digit to the **right of a decimal point**.	In the number 0.372, the 7 is in the second decimal place. 0.372 rounded to two decimal places is 0.37, because the 2 tells us to round down. Careful with money - don't write £27.4, instead write £27.40

1. Fractions and Decimals

14. Significant Figure	The significant figures of a number are the digits which **carry meaning** (ie. are significant) to the size of the number. The **first significant figure** of a number **cannot be zero**. In a number with a decimal, trailing zeros are not significant.	In the number 0.00821, the first significant figure is the 8. In the number 2.740, the 0 is not a significant figure. 0.00821 rounded to 2 significant figures is 0.0082. 19357 rounded to 3 significant figures is 19400. We need to include the two zeros at the end to keep the digits in the same place value columns.
14. Truncation	A method of approximating a decimal number by **dropping all decimal places** past a certain point **without rounding**.	3.14159265… can be truncated to 3.1415 (note that if it had been rounded, it would become 3.1416)
15. Error Interval	A **range of values** that a number could have taken before being rounded or truncated. An error interval is written using inequalities, with a **lower bound** and an **upper bound**. Note that the lower bound inequality can be 'equal to', but the upper bound cannot be 'equal to'.	0.6 has been rounded to 1 decimal place. The error interval is: $0.55 \leq x < 0.65$ The lower bound is 0.55 The upper bound is 0.65
16. Estimate	To find something **close to the correct answer**.	An estimate for the height of a man is 1.8 metres.
17. Approximation	When using approximations to estimate the solution to a calculation, **round each number in the calculation to 1 significant figure**. ≈ means 'approximately equal to'	$\frac{348+692}{0.526} \approx \frac{300+700}{0.5} = 2000$ 'Note that dividing by 0.5 is the same as multiplying by 2'
18. BIDMAS	An acronym for the **order** you should do calculations in. BIDMAS stands for **'Brackets, Indices, Division, Multiplication, Addition and Subtraction'**. Indices are also known as 'powers' or 'orders'. With strings of division and multiplication, or strings of addition and subtraction, and no brackets, work from left to right.	$6 + 3 \times 5 = 21, not\ 45$ $5^2 = 25$, where the 2 is the index/power. $12 \div 4 \div 2 = 1.5, not\ 6$

1. Fractions and Decimals

19. Recurring Decimal	A decimal number that has **digits that repeat forever**. The part that repeats is usually shown by placing a dot above the digit that repeats, or dots over the first and last digit of the repeating pattern.	$\frac{1}{3} = 0.333 \ldots = 0.\dot{3}$ $\frac{1}{7} = 0.142857142857 \ldots = 0.\dot{1}4285\dot{7}$ $\frac{77}{600} = 0.128333 \ldots = 0.128\dot{3}$
20. Rational Number	A number of the form $\frac{p}{q}$, where p **and** q **are integers** and $q \neq 0$. A number that cannot be written in this form is called an 'irrational' number	$\frac{4}{9}, 6, -\frac{1}{3}, \sqrt{25}$ are examples of rational numbers. $\pi, \sqrt{2}$ are examples of an irrational numbers.
21. Surd	The **irrational number** that is a **root of a positive integer,** whose value cannot be determined exactly. Surds have **infinite non-recurring decimals**.	$\sqrt{2}$ is a surd because it is a root which cannot be determined exactly. $\sqrt{2} = 1.41421356 \ldots$ which never repeats.
22. Fraction	A mathematical expression representing the **division** of one integer by another. Fractions are written as **two numbers separated by a horizontal line**.	$\frac{2}{7}$ is a 'proper' fraction. $\frac{9}{4}$ is an 'improper' or 'top-heavy' fraction.
23. Numerator	The **top** number of a fraction.	In the fraction $\frac{3}{5}$, 3 is the numerator.
24. Denominator	The **bottom** number of a fraction.	In the fraction $\frac{3}{5}$, 5 is the denominator.
25. Unit Fraction	A fraction where the **numerator is one** and the denominator is a positive integer.	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ $etc.$ are examples of unit fractions.
26. Reciprocal	The reciprocal of a number is **1 divided by the number**. The reciprocal of x is $\frac{1}{x}$ **When we multiply a number by its reciprocal, we get 1**. This is called the 'multiplicative inverse'.	The reciprocal of 5 is $\frac{1}{5}$ The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, because $\frac{2}{3} \times \frac{3}{2} = 1$
27. Mixed Number	A number formed of both an **integer part** and a **fraction part**.	$3\frac{2}{5}$ is an example of a mixed number.
28. Simplifying Fractions	**Divide the numerator and denominator by the highest common factor**.	$\frac{20}{45} = \frac{4}{9}$

1. Fractions and Decimals

29. Equivalent Fractions	Fractions which represent the **same value**.	$\frac{2}{5} = \frac{4}{10} = \frac{20}{50} = \frac{60}{150}\ etc.$
30. Comparing Fractions	To compare fractions, they each need to be rewritten so that they have a **common denominator**. **Ascending** means **smallest to biggest**. **Descending** means **biggest to smallest**.	Put in to ascending order: $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}, \frac{1}{2}$. Equivalent: $\frac{9}{12}, \frac{8}{12}, \frac{10}{12}, \frac{6}{12}$ Correct order: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$
31. Fraction of an Amount	**Divide** by the **bottom**, **times** by the **top**	Find $\frac{2}{5}$ of £60 $60 \div 5 = 12$ $12 \times 2 = 24$
32. Adding or Subtracting Fractions	Find the **LCM of the denominators** to find a common denominator. Use equivalent fractions to change each fraction to the **common denominator**. Then just **add or subtract the numerators** and keep the **denominator the same**.	$\frac{2}{3} + \frac{4}{5}$ Multiples of 3: 3, 6, 9, 12, **15**.. Multiples of 5: 5, 10, **15**.. LCM of 3 and 5 = 15 $\frac{2}{3} = \frac{10}{15}$ $\frac{4}{5} = \frac{12}{15}$ $\frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1\frac{7}{15}$
33. Multiplying Fractions	**Multiply** the **numerators** together and **multiply** the **denominators** together.	$\frac{3}{8} \times \frac{2}{9} = \frac{6}{72} = \frac{1}{12}$
34. Dividing Fractions	**‘Keep it, Flip it, Change it – KFC’** Keep the first fraction the same Flip the second fraction upside down Change the divide to a multiply Multiply by the reciprocal of the second fraction.	$\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$

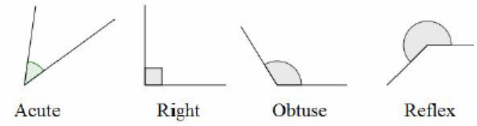
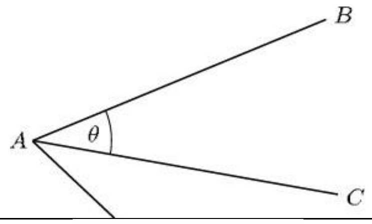
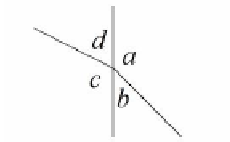
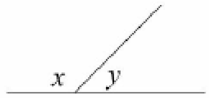
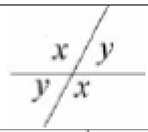
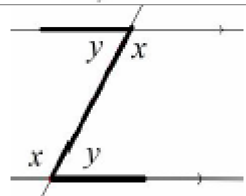
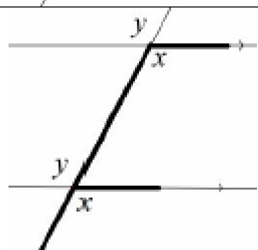
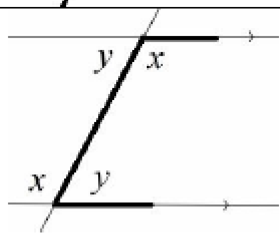
2. Expressions

Topic/Skill	Definition/Tips	Example
1. Expression	A mathematical statement written using **symbols**, **numbers** or **letters**,	3x + 2 or $5y^2$
2. Equation	A statement showing that **two expressions are equal** (i.e. has an equals sign)	2y – 17 = 15
3. Identity	An equation that is **true for all values** of the variables An identity uses the symbol: ≡	$2x \equiv x+x$
4. Formula	Shows the **relationship** between **two or more variables**	Area of a rectangle = length x width or A= LxW
5. Simplifying Expressions	**Collect 'like terms'.** Be careful with negatives. x^2 and x are not like terms.	$2x + 3y + 4x - 5y + 3$ $= 6x - 2y + 3$ $3x + 4 - x^2 + 2x - 1 = 5x - x^2 + 3$
6. x times x	The answer is x^2 not $2x$.	Squaring is multiplying by itself, not by 2.
7. $p \times p \times p$	The answer is p^3 not $3p$	If p=2, then p^3=2x2x2=8, not 2x3=6
8. $p + p + p$	The answer is 3p not p^3	If p=2, then 2+2+2=6, not $2^3 = 8$
9. Expand	To expand a bracket, **multiply** each term **in the bracket** by the expression **outside** the bracket.	$3(m + 7) = 3x + 21$ $(x + 5)(x + 2) = x^2 + 7x + 10$
10. Factorise	The **reverse** of **expanding**. Factorising is writing an expression as a product of terms by '**taking out' a common factor** and putting in bracket(s).	$6x - 15 = 3(2x - 5)$, where 3 is the common factor. $x^2 + 8x + 12 = (x + 6)(x + 2)$
11. Quadratic	A quadratic expression is of the form $\boldsymbol{ax^2 + bx + c}$ where a, b and c are numbers, $\boldsymbol{a \neq 0}$	Examples of quadratic expressions: x^2 $8x^2 - 3x + 7$ Examples of non-quadratic expressions: $2x^3 - 5x^2$ $9x - 1$
12. Factorising Quadratics	When a quadratic expression is in the form $x^2 + bx + c$ find the two numbers that **add to give b** and **multiply to give c**.	$x^2 + 7x + 10 = (x + 5)(x + 2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2 + 2x - 8 = (x + 4)(x - 2)$ (because +4 and -2 add to give +2 and multiply to give -8)

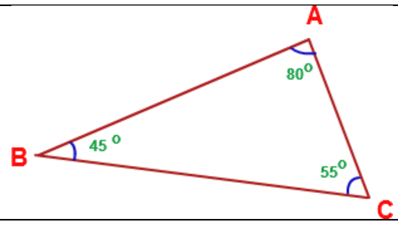
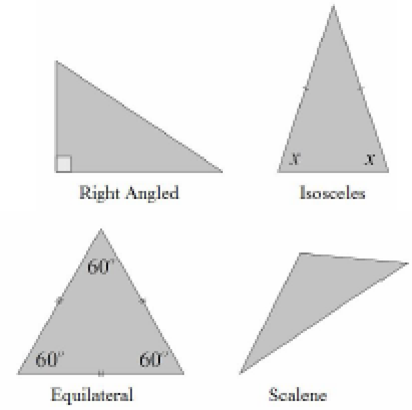
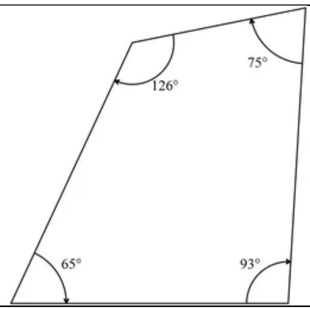
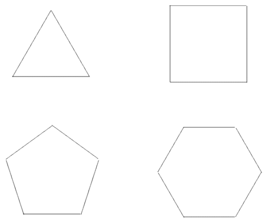
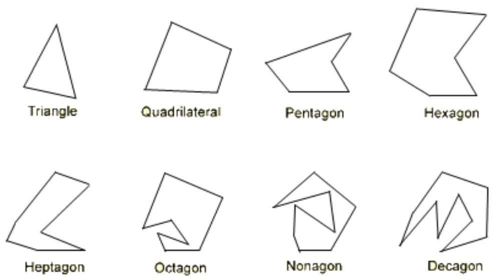
2. Expressions

13. Difference of Two Squares	An expression of the form $\boldsymbol{a^2 - b^2}$ can be factorised to give $(\boldsymbol{a+b})(\boldsymbol{a-b})$	$x^2 - 25 = (x+5)(x-5)$ $16x^2 - 81 = (4x+9)(4x-9)$
14. Solving Quadratics ($\boldsymbol{ax^2 = b}$)	Isolate the x^2 term and square root both sides. Remember there will be a **positive and a negative solution**.	$2x^2 = 98$ $x^2 = 49$ $x = \pm 7$
15. Solving Quadratics ($\boldsymbol{ax^2 + bx = 0}$)	**Factorise** and then **solve = 0**.	$x^2 - 3x = 0$ $x(x-3) = 0$ $x = 0 \text{ or } x = 3$
16. Solving Quadratics by Factorising ($\boldsymbol{a = 1}$)	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $x^2 + 3x - 10 = 0$ Factorise: $(x+5)(x-2) = 0$ $x = -5 \text{ or } x = 2$
17. Factorising Quadratics when $\boldsymbol{a \neq 1}$	When a quadratic is in the form $ax^2 + bx + c$ 1. Multiply a by c = ac 2. Find two numbers that add to give b and multiply to give ac. 3. Re-write the quadratic, replacing bx with the two numbers you found. 4. Factorise in pairs – you should get the same bracket twice 5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.	Factorise $6x^2 + 5x - 4$ 1. $6 \times -4 = -24$ 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 3. $6x^2 + 8x - 3x - 4$ 4. Factorise in pairs: $2x(3x+4) - 1(3x+4)$ 5. Answer = $(3x+4)(2x-1)$
18. Solving Quadratics by Factorising ($\boldsymbol{a \neq 1}$)	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $2x^2 + 7x - 4 = 0$ Factorise: $(2x-1)(x+4) = 0$ $x = \frac{1}{2} \text{ or } x = -4$

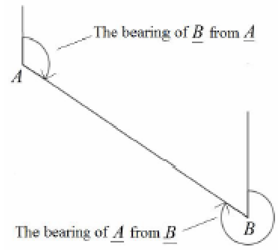
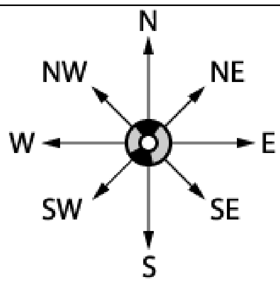
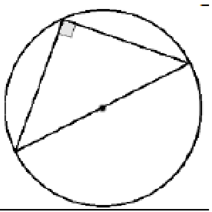
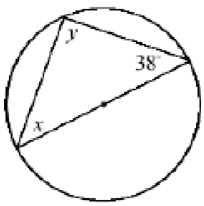
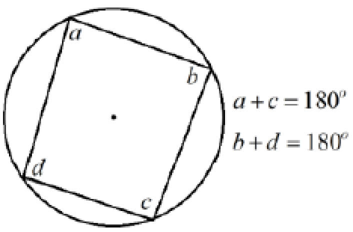
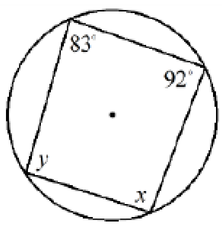
3. Angles and Circle Theorems

Topic/Skill	Definition/Tips	Example
1. Types of Angles	**Acute angles** are less than 90°. **Right angles** are exactly 90°. **Obtuse angles** are greater than 90° but less than 180°. **Reflex angles** are greater than 180° but less than 360°.	
2. Angle Notation	Can use **one lower-case** letters, eg. θ or x Can use **three upper-case** letters, eg. Angle BAC, or $B\hat{A}C$	
3. Angles at a Point	**Angles around a point add up to 360°**.	 $a+b+c+d=360^o$
4. Angles on a Straight Line	**Angles around a point on a straight line add up to 180°**.	 $x+y=180^o$
5. Opposite Angles	**Vertically opposite angles are equal**.	
6. Alternate Angles	**Alternate angles are equal**. Look for the Z shape (forwards or backwards).	
7. Corresponding Angles	**Corresponding angles are equal**. Look for the F shape (in any direction).	
8. Co-Interior Angles	**Co-Interior angles add up to 180°**. Look for the C shape	

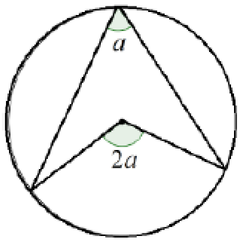
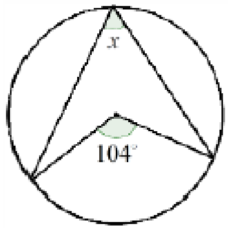
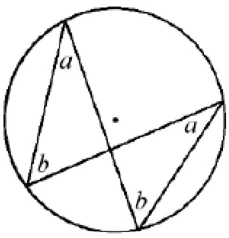
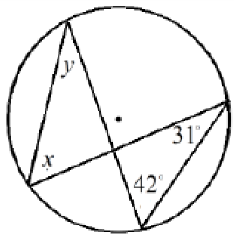
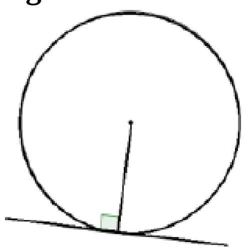
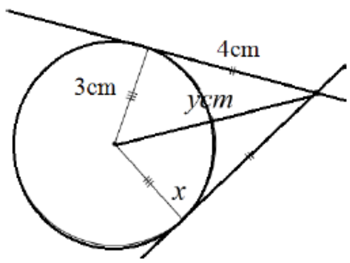
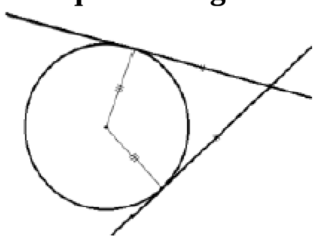
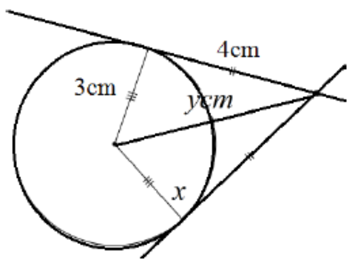
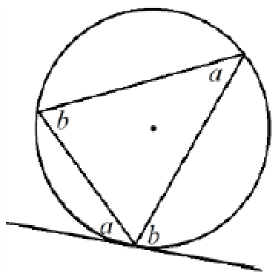
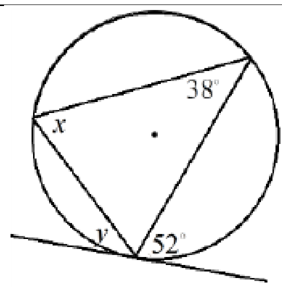
3. Angles and Circle Theorems

9. Angles in a Triangle	**Angles in a triangle add up to 180°.**	
10. Types of Triangles	**Right Angle** Triangles have a **90°** angle in. **Isosceles** Triangles have **2 equal sides** and **2 equal base angles**. **Equilateral** Triangles have **3 equal sides** and **3 equal angles (60°)**. **Scalene** Triangles have **different sides** and **different angles**. **Base angles in an isosceles triangle are equal.**	
11. Angles in a Quadrilateral	**Angles in a quadrilateral add up to 360°.**	
12. Polygon	A **2D** shape with **only straight edges**.	Rectangle, Hexagon, Decagon, Kite etc.
13. Regular	A shape is **regular** if all the **sides** and all the **angles** are **equal**.	
14. Names of Polygons	**3**-sided = **Triangle** **4**-sided = **Quadrilateral** **5**-sided = **Pentagon** **6**-sided = **Hexagon** **7**-sided = **Heptagon**/Septagon **8**-sided = **Octagon** **9**-sided = **Nonagon** **10**-sided = **Decagon**	
15. Sum of Interior Angles	$(n-2)\times 180$ where n is the number of sides.	Sum of Interior Angles in a Decagon = $(10-2)\times 180 = 1440°$

3. Angles and Circle Theorems

16. Size of Interior Angle in a Regular Polygon	$\frac{(n-2)\times 180}{n}$ You can also use the formula: $180 - Size\ of\ Exterior\ Angle$	Size of Interior Angle in a Regular Pentagon = $\frac{(5-2)\times 180}{5} = 108°$
17. Size of Exterior Angle in a Regular Polygon	$\frac{360}{n}$ You can also use the formula: $180 - Size\ of\ Interior\ Angle$	Size of Exterior Angle in a Regular Octagon = $\frac{360}{8} = 45°$
18. Bearings	1. Measure from **North** (draw a North line) 2. Measure **clockwise** 3. Your answer must have **3 digits** (eg. 047°) Look out for where the bearing is measured <u>from</u>.	
19. Compass Directions	You can use an acronym such as **'Never Eat Shredded Wheat'** to remember the order of the compass directions in a clockwise direction. Bearings: $NE = 045°, W = 270°\ etc.$	
Circle Theorem 1	**Angles in a semi-circle are 90°** 	 $y = 90°$ $x = 180 - 90 - 38 = 52°$
Circle Theorem 2	**Opposite angles in a cyclic quadrilateral add up to 180°.** 	 $x = 180 - 83 = 97°$ $y = 180 - 92 = 88°$

3. Angles and Circle Theorems

Circle Theorem 3	**The angle at the centre is twice the angle at the circumference.** 	 $x = 104 \div 2 = 52°$
Circle Theorem 4	**Angles in the same segment are equal.** 	 $x = 42°$ $y = 31°$
Circle Theorem 5	**A tangent meets a radius at a right angle.** 	 $y = 5cm$ (Pythagoras' Theorem)
Circle Theorem 6	**Two tangents from an external point are equal in length.** 	 $x = 90°$
Circle Theorem 7	**Alternate Segment Theorem** 	 $x = 52°$ $y = 38°$

4. Handling Data

<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Types of Data</td><td>Qualitative Data – non-numerical data
Quantitative Data – numerical data

Continuous Data – data that can take any numerical value within a given range.
Discrete Data – data that can take only specific values within a given range.</td><td>Qualitative Data – eye colour, gender etc.

Continuous Data – weight, voltage etc.

Discrete Data – number of children, shoe size etc.</td></tr>
<tr><td>2. Grouped Data</td><td>Data that has been bundled in to categories.

Seen in grouped frequency tables, histograms, cumulative frequency etc.</td><td><table><tr><th>Foot length, l, (cm)</th><th>Number of children</th></tr><tr><td>$10 \leqslant l < 12$</td><td>5</td></tr><tr><td>$12 \leqslant l < 17$</td><td>53</td></tr></table></td></tr>
<tr><td>3. Primary /Secondary Data</td><td>Primary Data – collected yourself for a specific purpose.

Secondary Data – collected by someone else for another purpose.</td><td>Primary Data – data collected by a student for their own research project.

Secondary Data – Census data used to analyse link between education and earnings.</td></tr>
<tr><td>4. Mean</td><td>Add up the values and divide by how many values there are.</td><td>The mean of 3, 4, 7, 6, 0, 4, 6 is
$\frac{3+4+7+6+0+4+6}{7} = 5$</td></tr>
<tr><td>5. Mean from a Table</td><td>1. Find the midpoints (if necessary)
2. Multiply Frequency by values or midpoints
3. Add up these values
4. Divide this total by the Total Frequency

If grouped data is used, the answer will be an estimate. (The use of the word ‘estimate’ here does not mean round everything to 1 significant figure)</td><td><table><tr><th>Height in cm</th><th>Frequency</th><th>Midpoint</th><th>F × M</th></tr><tr><td>$0 < h \leq 10$</td><td>8</td><td>5</td><td>8×5=40</td></tr><tr><td>$10 < h \leq 30$</td><td>10</td><td>20</td><td>10×20=200</td></tr><tr><td>$30 < h \leq 40$</td><td>6</td><td>35</td><td>6×35=210</td></tr><tr><td>Total</td><td>24</td><td>Ignore!</td><td>450</td></tr></table>Estimated Mean height: 450 ÷ 24 = 18.75cm</td></tr>
<tr><td>6. Median Value</td><td>The middle value.

Put the data in order and find the middle one.
If there are two middle values, find the number half way between them by adding them together and dividing by 2.</td><td>Find the median of: 4, 5, 2, 3, 6, 7, 6

Ordered: 2, 3, 4, 5, 6, 6, 7

Median = 5</td></tr>
<tr><td>7. Median from a Table</td><td>Use the formula $\frac{(n+1)}{2}$ to find the position of the median.

n is the total frequency.</td><td>If the total frequency is 15, the median will be the $\left(\frac{15+1}{2}\right) = 8th$ position</td></tr>
</table>

4. Handling Data

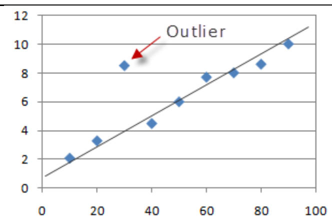
<table>
<tr><td>8. Mode /Modal Value</td><td>Most frequent/common.

Can have more than one mode (called bi-modal or multi-modal) or no mode (if all values appear once)</td><td>Find the mode: 4, 5, 2, 3, 6, 4, 7, 8, 4

Mode = 4</td></tr>
<tr><td>9. Range</td><td>Highest value subtract the Smallest value

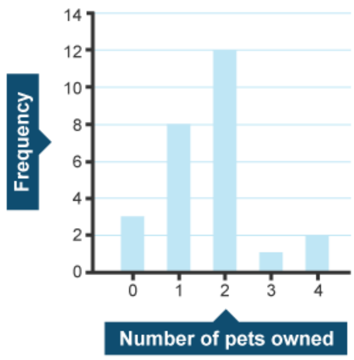
Range is a ‘measure of spread’. The smaller the range the more <u>consistent</u> the data, the wider the range, the <u>less consistent</u> or <u>more variable</u> the data.</td><td>Find the range: 3, 31, 26, 102, 37, 97.

Range = 102-3 = 99</td></tr>
<tr><td>10. Outlier</td><td>A value that ‘lies outside’ most of the other values in a set of data.
An outlier is much smaller or much larger than the other values in a set of data.</td><td>

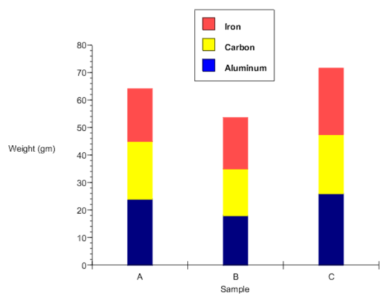


</td></tr>
<tr><td>11. Frequency Table</td><td>A record of how often each value in a set of data occurs.</td><td>
<table>
<tr><th>Number of marks</th><th>Tally marks</th><th>Frequency</th></tr>
<tr><td>1</td><td>卌 II</td><td>7</td></tr>
<tr><td>2</td><td>卌</td><td>5</td></tr>
<tr><td>3</td><td>卌 I</td><td>6</td></tr>
<tr><td>4</td><td>卌</td><td>5</td></tr>
<tr><td>5</td><td>III</td><td>3</td></tr>
<tr><td>Total</td><td></td><td>26</td></tr>
</table>
</td></tr>
<tr><td>12. Bar Chart</td><td>Represents data as vertical blocks.

$x-axis$ shows the type of data
$y-axis$ shows the frequency for each type of data
Each bar should be the same width
There should be gaps between each bar
Remember to label each axis.</td><td>



</td></tr>
<tr><td>13. Types of Bar Chart</td><td>Compound/Composite Bar Charts show data stacked on top of each other.</td><td>

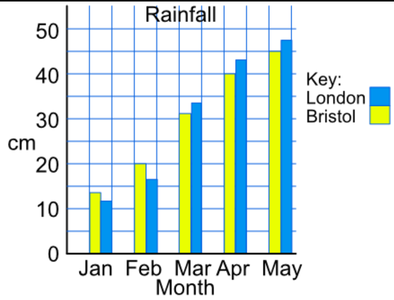


</td></tr>
</table>

4. Handling Data

Comparative/Dual Bar Charts show data side by side.

Make sure you have a clear **key**.



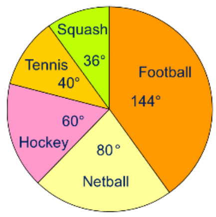
Dual Bar Chart

14. Pie Chart

Used for showing **how data breaks down into** its constituent **parts**.

When drawing a pie chart, **divide 360° by the total frequency**. This will tell you how many degrees to use for the frequency of each category.

Remember to **label** the category that each sector in the pie chart represents.

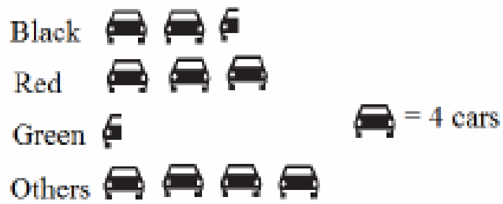


If there are 40 people in a survey, then each person will be worth 360÷40=9° of the pie chart.

15. Pictogram

Uses **pictures** or symbols to **show the value** of the data.

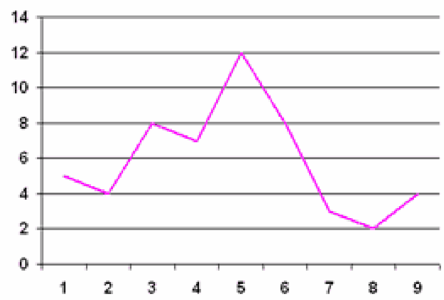
A pictogram must have a **key**.



16. Line Graph

A graph that uses **points connected by straight lines** to show how data changes in values.

This can be used for **time series data**, which is a series of data points spaced over uniform time intervals in **time order**.



17. Two Way Tables

A table that **organises data** around **two categories.**

Fill out the information step by step using the information given.

Make sure all the totals add up for all columns and rows.

Question: Complete the 2 way table below.

	Left Handed	Right Handed	**Total**
Boys	10		58
Girls			
Total		84	100

Answer: Step 1, fill out the easy parts (the totals)

	Left Handed	Right Handed	**Total**
Boys	10	*48*	58
Girls			*42*
Total	*16*	84	100

Answer: Step 2, fill out the remaining parts

	Left Handed	Right Handed	**Total**
Boys	10	*48*	58
Girls	*6*	*36*	*42*
Total	*16*	84	100

18. Correlation

Correlation between two sets of data means they are **connected** in some way.

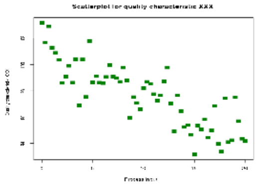
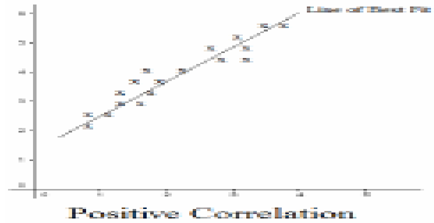
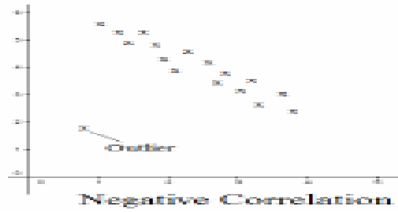
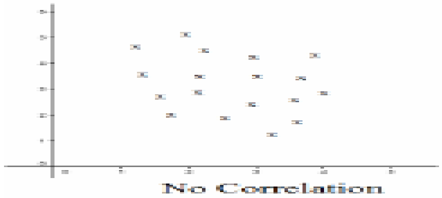
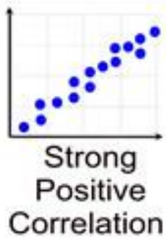
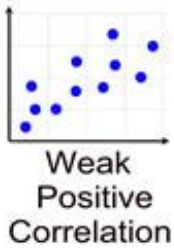
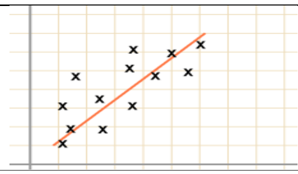
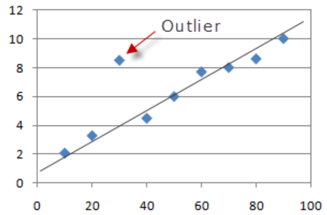
There is correlation between temperature and the number of ice creams sold.

19. Causality

When one variable **influences** another variable.

The more hours you work at a particular job (paid hourly), the higher your income from that job will be.

4. Handling Data

20. Scatter Graph	A graph in which values of **two variables** are plotted along two axes to **compare** them and see if there is any **connection** between them.	
21. Positive Correlation	As one value **increases** the other value **increases**.	
22. Negative Correlation	As one value **increases** the other value **decreases**.	
23. No Correlation	There is **no linear relationship** between the two.	
24. Strong Correlation	When two sets of data are **closely linked**.	
25. Weak Correlation	When two sets of data have correlation, but are **not closely linked**.	
26. Line of Best Fit	A **straight line** that **best represents the data** on a scatter graph. Note: The line does not have to start at the origin.	
27. Outlier	A value that 'lies outside' most of the other values in a set of data. An outlier is **much smaller or much larger** than the other values in a set of data.	

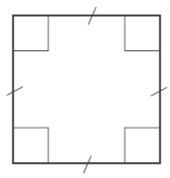
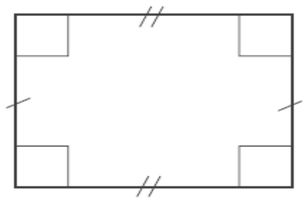
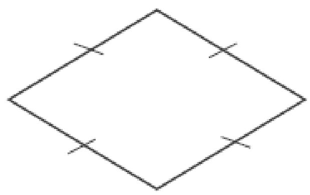
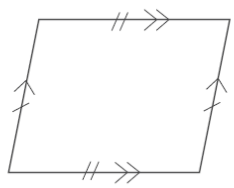
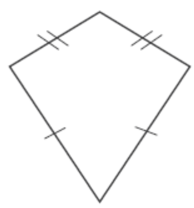
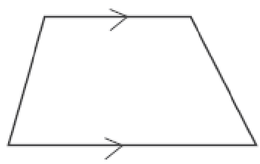
5. Percentages

Topic/Skill	Definition/Tips	Example
1. Percentage	**Number of parts per 100.**	31% means $\frac{31}{100}$
2. Finding 10%	To find **10%**, **divide by 10**	10% of £36 = 36÷10=£3.60
3. Finding 1%	To find **1%**, **divide by 100**	1% of £8 = 8÷100 = £0.08
4. Percentage Change	$\frac{\boldsymbol{Difference}}{\boldsymbol{Original}} \times \mathbf{100}\%$	A games console is bought for £200 and sold for £250. % change = $\frac{50}{200} \times 100 = 25\%$
5. Fractions to Decimals	**Divide the numerator by the denominator** using the bus stop method.	$\frac{3}{8} = 3 \div 8 = 0.375$
6. Decimals to Fractions	**Write as a fraction** over 10, 100 or 1000 and simplify.	$0.36 = \frac{36}{100} = \frac{9}{25}$
7. Percentages to Decimals	**Divide by 100**	$8\% = 8 \div 100 = 0.08$
8. Decimals to Percentages	**Multiply by 100**	$0.4 = 0.4 \times 100\% = 40\%$
9. Fractions to Percentages	Percentage is just a fraction out of 100. **Make the denominator 100 using equivalent fractions**. When the denominator doesn't go in to 100, use a calculator and **multiply the fraction by 100**.	$\frac{3}{25} = \frac{12}{100} = 12\%$ $\frac{9}{17} \times 100 = 52.9\%$
10. Percentages to Fractions	Percentage is just a fraction out of 100. **Write the percentage over 100** and simplify.	$14\% = \frac{14}{100} = \frac{7}{50}$
11. Increase or Decrease by a Percentage	Non-calculator: **Find the percentage** and **add** or **subtract** it from the **original** amount. Calculator: Find the **percentage multiplier** and multiply.	<u>Increase 500 by 20% (Non Calc):</u> 10% of 500 = 50 so 20% of 500 = 100 500 + 100 = 600 <u>Decrease 800 by 17% (Calc):</u> 100%-17%=83% 83% ÷ 100 = 0.83 0.83 x 800 = 664

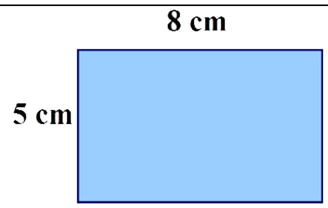
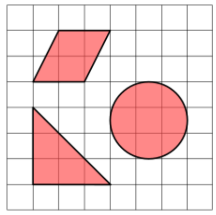
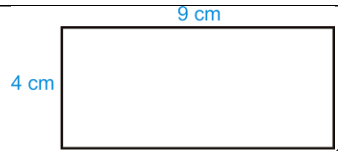
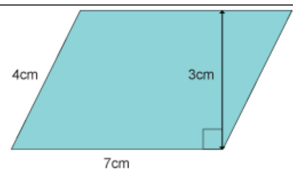
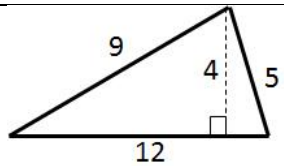
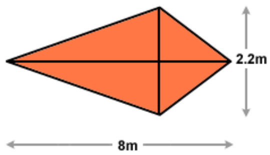
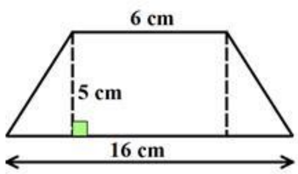
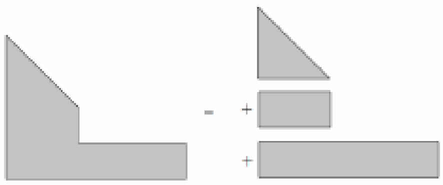
5. Percentages

12. Percentage Multiplier	The **number** you **multiply** a quantity by to **increase or decrease** it by a **percentage**.	The multiplier for increasing by 12% is 1.12 The multiplier for decreasing by 12% is 0.88 The multiplier for increasing by 100% is 2.
13. Reverse Percentage	Find the **correct percentage given in the question**, then work backwards to **find 100%** Look out for words like '**before**' or '**original**'	A jumper was priced at £48.60 after a 10% reduction. Find its original price. 100% - 10% = 90% 90% = £48.60 1% = £0.54 100% = £54
14. Simple Interest	Interest calculated as a **percentage of the original** amount.	£1000 invested for 3 years at 5% simple interest. 5% of £1000 = £50 Interest = $3 \times £50 = £150$ Balance = £1150
15. Compound Interest	Interest is calculated on the new balance each step (e.g. per year). Use percentage multipliers raised to the power of how many 'steps' are needed.	£1000 invested for 3 years at 5% compound interest Multiplier for increasing by 5% is 1.05 1000 x 1.05^3 = £1157.63 (Balance) 1157.63-1000 = £157.63 (Interest)
16. Exponential Growth	When we **multiply** a number **repeatedly** by the **same number** ($\neq 1$), resulting in the number **increasing by the same proportion** each time. The original amount can grow very quickly in exponential growth.	$1, 2, 4, 8, 16, 32, 64, 128$ ... is an example of exponential growth, because the numbers are being multiplied by 2 each time.
17. Exponential Decay	When we **multiply** a number **repeatedly** by the **same number** ($\mathbf{0 < x < 1}$), resulting in the number **decreasing by the same proportion** each time. The original amount can decrease very quickly in exponential decay.	$1000, 200, 40, 8$ ... is an example of exponential decay, because the numbers are being multiplied by $\frac{1}{5}$ each time.
18. Compound Interest	Interest paid on the **original amount and the accumulated interest**.	A bank pays 5% compound interest a year. Bob invests £3000. How much will he have after 7 years. $3000 \times 1.05^7 = £4221.30$

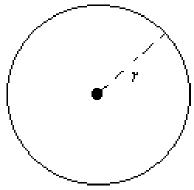
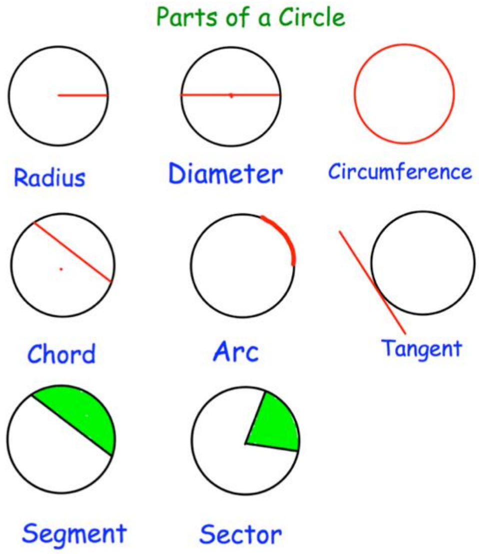
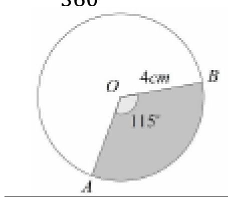
6. 2D Shapes

Topic/Skill	Definition/Tips	Example
1. Square	• **Four equal sides** • **Four right angles** • **Opposite sides parallel** • **Diagonals bisect** each other at **right angles** • **Four lines** of **symmetry** • **Rotational symmetry** of **order four**	
2. Rectangle	• **Two pairs** of **equal sides** • **Four right angles** • **Opposite sides parallel** • **Diagonals bisect** each other**, not at right angles** • **Two lines** of **symmetry** • **Rotational symmetry** of **order two**	
3. Rhombus	• **Four equal sides** • **Diagonally opposite angles are equal** • **Opposite sides parallel** • **Diagonals bisect** each other at **right angles** • **Two lines** of **symmetry** • **Rotational symmetry** of **order two**	
4. Parallelogram	• **Two pairs** of **equal sides** • **Diagonally opposite angles are equal** • **Opposite sides parallel** • **Diagonals bisect** each other**, not at right angles** • **No lines** of **symmetry** • **Rotational symmetry** of **order two**	
5. Kite	• **Two pairs** of **adjacent sides** of **equal** length • **One pair** of **diagonally opposite angles are equal** (where different length sides meet) • **Diagonals intersect** at **right angles, but do not bisect** • **One line** of **symmetry** • **No rotational symmetry**	
6. Trapezium	• **One pair** of **parallel sides** • **No lines of symmetry** • **No rotational symmetry** Special Case: Isosceles Trapeziums have one line of symmetry.	

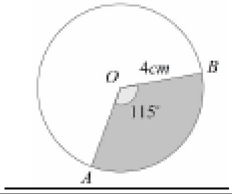
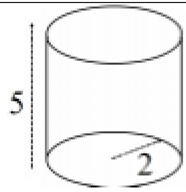
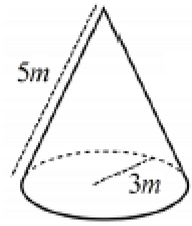
6. 2D Shapes

7. Perimeter	The **total distance** around the **outside** of a shape. Units simply represent a length: mm, cm, m etc.	 $P = 8 + 5 + 8 + 5 = 26cm$
8. Area	The amount of **space inside** a shape. Units are now squared to represent 2 dimensions being involved: mm^2, cm^2, m^2	
9. Area of a rectangle	**Length x Width**	 $A = 36cm^2$
10. Area of a Parallelogram	**Base x Perpendicular Height** Not the sloping height.	 $A = 21cm^2$
11. Area of a Triangle	$\frac{1}{2} \times$ **Base x Height**	 $A = 24cm^2$
12. Area of a Kite	Split in to **two triangles** and use the method above.	 $A = 8.8m^2$
13. Area of a Trapezium	$$\frac{(a+b)}{2} \times h$$ "Half the sum of the parallel side, times the height between them. That is how you calculate the area of a trapezium"	 $A = 55cm^2$
14. Compound Shape	A shape made up of a **combination of other known shapes** put together.	

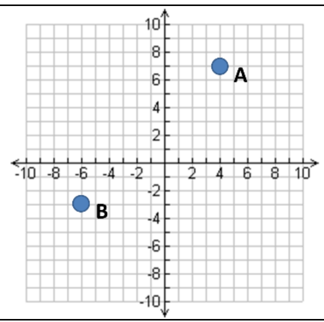
6. 2D Shapes

15. Circle	A circle is the locus of all points equidistant from a central point.	
16. Parts of a Circle	**Radius** – the **distance** from the **centre** of a circle to the **edge** **Diameter** – the total **distance** across the **width** of a circle **through the centre**. **Circumference** – the **total distance** around the **outside** of a circle **Chord** – a **straight line** whose **end points lie on a circle** **Tangent** – a **straight line** which **touches** a circle at exactly **one point** **Arc** – a **part of the circumference** of a circle **Sector** – the **region** of a circle enclosed by **two radii** and their intercepted **arc** **Segment** – the **region** bounded by a **chord** and the **arc** created by the chord	
17. Area of a Circle	$\boldsymbol{A = \pi r^2}$ which means 'pi x radius squared'.	If the radius was 5cm, then: $A = \pi \times 5^2 = 78.5cm^2$
18. Circumference of a Circle	$\boldsymbol{C = \pi d}$ which means 'pi x diameter'	If the radius was 5cm, then: $C = \pi \times 10 = 31.4cm$
19. π ('pi')	Pi is the circumference of a circle divided by the diameter. $\boldsymbol{\pi \approx 3.14}$	
20. Arc Length of a Sector	The arc length is a fraction of the full circumference. Take the **angle** given **as a fraction over 360°** and **multiply** by the **circumference**.	Arc Length = $\frac{115}{360} \times \pi \times 8 = 8.03cm$ 

6. 2D Shapes

21. Area of a Sector	The area of a sector is a fraction of the full circle area. Take the **angle** given **as a fraction over 360°** and **multiply** by the **area**.	Area = $\frac{115}{360} \times \pi \times 4^2 = 16.1cm^2$ 
22. Surface Area of a Cylinder	**Curved Surface Area =** $\boldsymbol{\pi dh}$ or $\boldsymbol{2\pi rh}$ **Total SA =** $\boldsymbol{2\pi r^2 + \pi dh}$ or $\boldsymbol{2\pi r^2 + 2\pi rh}$	 $Total\ SA = 2\pi(2)^2 + \pi(4)(5) = 28\pi$
23. Surface Area of a Cone	**Curved Surface Area =** $\boldsymbol{\pi rl}$ where $l = slant\ height$ **Total SA =** $\boldsymbol{\pi rl + \pi r^2}$ You may need to use Pythagoras' Theorem to find the slant height	 $Total\ SA = \pi(3)(5) + \pi(3)^2 = 24\pi$
24. Surface Area of a Sphere	$\boldsymbol{SA = 4\pi r^2}$ Look out for hemispheres – halve the SA of a sphere and add on a circle (πr^2)	Find the surface area of a sphere with radius 3cm. $SA = 4\pi(3)^2 = 36\pi cm^2$

7. Linear Equations and Graphs

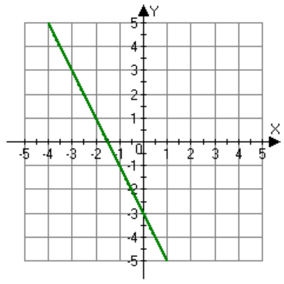
Topic/Skill	Definition/Tips	Example
1. Solve	To find the **answer**/value of something **Use inverse operations** on both sides of the equation (balancing method) until you find the value for the letter.	Solve $2x - 3 = 7$ *Add 3 on both sides* $2x = 10$ *Divide by 2 on both sides* $x = 5$
2. Inverse	**Opposite**	The inverse of addition is subtraction. The inverse of multiplication is division. The inverse of square is square root.
3. Rearranging Formulae	**Use inverse operations** on both sides of the formula (balancing method) until you find the expression for the letter.	Make x the subject of $y = \frac{2x-1}{z}$ *Multiply both sides by z* $yz = 2x - 1$ *Add 1 to both sides* $yz + 1 = 2x$ *Divide by 2 on both sides* $\frac{yz+1}{2} = x$ We now have x as the subject.
4. Writing Formulae	**Substitute letters for words** in the question.	Bob charges £3 per window and a £5 call out charge. $C = 3N + 5$ Where N=number of windows and C=cost
5. Substitution	**Replace letters with numbers**. Be careful of $5x^2$. You need to square first, then multiply by 5.	$a = 3, b = 2$ *and* $c = 5$. Find: 1. $2a = 2 \times 3 = 6$ 2. $3a - 2b = 3 \times 3 - 2 \times 2 = 5$ 3. $7b^2 - 5 = 7 \times 2^2 - 5 = 23$
6. Coordinates	Written in **pairs**. The **first** term is the **x-coordinate** (movement **across**). The **second** term is the **y-coordinate** (movement **up or down**)	 A: (4,7) B: (-6,-3)
7. Midpoint of a Line	Method 1: **add the x coordinates and divide by 2, add the y coordinates and divide by 2** Method 2: Sketch the line and find the values half way between the two x and two y values.	Find the midpoint between (2,1) and (6,9) $\frac{2+6}{2} = 4$ and $\frac{1+9}{2} = 5$ So, the midpoint is (4,5)

7. Linear Equations and Graphs

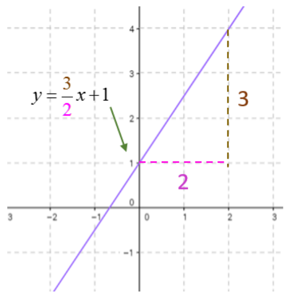
<table>
<tr>
<td>8. Linear Graph</td>
<td>Straight line graph.

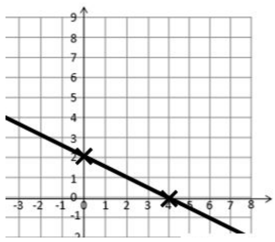
The general equation of a linear graph is
$\boldsymbol{y = mx + c}$

where $\boldsymbol{m}$ is the gradient and c is the y-intercept.

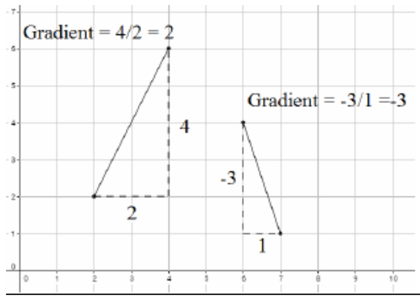
The equation of a linear graph can contain an x-term, a y-term and a number.</td>
<td>Example:

Other examples:
$x = y$
$y = 4$
$x = -2$
$y = 2x - 7$
$y + x = 10$
$2y - 4x = 12$</td>
</tr>
<tr>
<td>9. Plotting Linear Graphs</td>
<td>Method 1: Table of Values
Construct a table of values to calculate coordinates.

Method 2: Gradient-Intercept Method
(use when the equation is in the form $y = mx + c$)
1. Plots the y-intercept
2. Using the gradient, plot a second point.
3. Draw a line through the two points plotted.

Method 3: Cover-Up Method (use when the equation is in the form $ax + by = c$)
1. Cover the x term and solve the resulting equation. Plot this on the $x - axis$.
2. Cover the y term and solve the resulting equation. Plot this on the $y - axis$.
3. Draw a line through the two points plotted.</td>
<td><table><tr><td>x</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>y= x +3</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr></table>


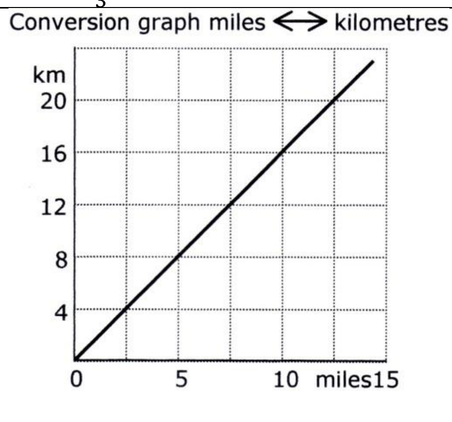

$2x + 4y = 8$</td>
</tr>
<tr>
<td>10. Gradient</td>
<td>The gradient of a line is how steep it is.

Gradient =
$\boldsymbol{\frac{Change\ in\ y}{Change\ in\ x} = \frac{Rise}{Run}}$

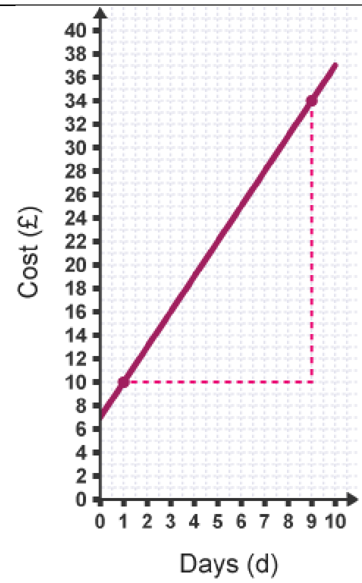
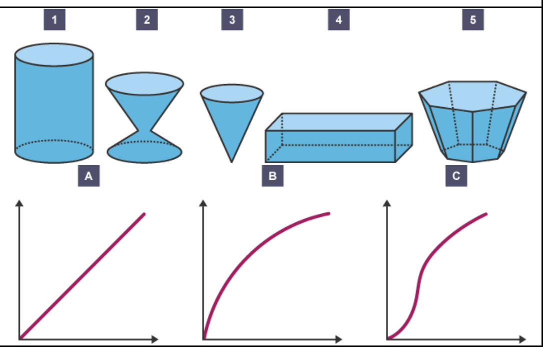
The gradient can be positive (sloping upwards) or negative (sloping downwards)</td>
<td>
</td>
</tr>
<tr>
<td>11. Finding the Equation of a Line <u>given a point and a gradient</u></td>
<td>Substitute in the gradient (m) and point (x,y) in to the equation $\boldsymbol{y = mx + c}$ and solve for c.</td>
<td>Find the equation of the line with gradient 4 passing through (2,7).

$y = 4x + c$
$7 = 4 \times 2 + c$
$c = -1$
$y = 4x - 1$</td>
</tr>
</table>

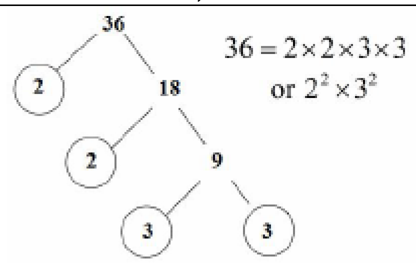
7. Linear Equations and Graphs

12. Finding the Equation of a Line <u>given two points</u>	Use the two points to **calculate the gradient**. Then **repeat the method above** using the gradient and either of the points.	Find the equation of the line passing through (6,11) and (2,3) $m = \frac{11-3}{6-2} = 2$ $y = 2x + c$ $11 = 2 \times 6 + c$ $c = -1$ $y = 2x - 1$
13 Parallel Lines	If two lines are **parallel**, they will have the **same gradient**. The value of m will be the same for both lines. You may need to rearrange equations of lines to compare gradients (they need to be in the form $y = mx + c$)	Are the lines $y = 3x - 1$ and $2y - 6x + 10 = 0$ parallel? Firstly, rearrange the second equation in to the form $y = mx + c$ $2y - 6x + 10 = 0 \rightarrow y = 3x - 5$ Since the two gradients are equal (3), the lines are parallel.
14. Perpendicular Lines	If two lines are **perpendicular**, the **product** of their **gradients** will always equal **-1**. The gradient of one line will be the **negative reciprocal** of the gradient of the other line. You may need to rearrange equations of lines to understand the gradient (they need to be in the form $y = mx + c$)	Find the equation of the line perpendicular to $y = 3x + 2$ which passes through (6,5) As they are perpendicular, the gradient of the new line will be $-\frac{1}{3}$ as this is the negative reciprocal of 3. $y = -\frac{1}{3}x + c$ $5 = -\frac{1}{3} \times 6 + c$ $c = 7$ $y = -\frac{1}{3}x + 7$ or $3x + x - 7 = 0$
15. Conversion Graph	A line graph to **convert one unit to another**. Can be used to convert units (eg. miles and kilometres) or currencies ($ and £) Find the value you know on one axis, read up/across to the conversion line and read the equivalent value from the other axis.	 $8\ km = 5\ miles$

7. Linear Equations and Graphs

16. Real Life Graphs	Graphs that are supposed to model some real-life situation. The actual meaning of the values depends on the labels and units on each axis. The **gradient** might have a contextual meaning. The **y-intercept** might have a contextual meaning. The **area** under the graph might have a contextual meaning.	 A graph showing the cost of hiring a ladder for various numbers of days. The gradient shows the cost per day. It costs £3/day to hire the ladder. The y-intercept shows the additional cost/deposit/fixed charge (something not linked to how long the ladder is hired for). The additional cost is £7.
17. Depth of Water in Containers	Graphs can be used to show how the depth of water changes as different shaped containers are filled with water at a constant rate.	

8. Number

Topic/Skill	Definition/Tips	Example
1. Multiple	The result of multiplying a number by an integer. The **times tables** of a number.	The first five multiples of 7 are: $7, 14, 21, 28, 35$
2. Factor	A number that **divides exactly** into another number without a remainder. It is useful to write factors in pairs	The factors of 18 are: $1, 2, 3, 6, 9, 18$ The factor pairs of 18 are: $1, 18$ $2, 9$ $3, 6$
3. Lowest Common Multiple (LCM)	The **smallest** number that is in the **times tables** of each of the numbers given.	The LCM of 3, 4 and 5 is 60 because it is the smallest number in the 3, 4 and 5 times tables.
4. Highest Common Factor (HCF)	The **biggest** number that **divides exactly** into two or more numbers.	The HCF of 6 and 9 is 3 because it is the biggest number that divides into 6 and 9 exactly.
5. Prime Number	A number with **exactly two factors**. A number that can only be divided by itself and one. The number **1 is not prime**, as it only has one factor, not two.	The first ten prime numbers are: $2, 3, 5, 7, 11, 13, 17, 19, 23, 29$
6. Prime Factor	A factor which is a prime number.	The prime factors of 18 are: $2, 3$
7. Product of Prime Factors	Finding out which **prime numbers multiply** together to make the **original** number. Use a **prime factor tree.** Also known as 'prime factorisation'.	
8. Square Number	The number you get when you **multiply a number by itself**.	**1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225…** $9^2 = 9 \times 9 = 81$
9. Square Root	The **number you multiply by itself** to get another number. The reverse process of squaring a number.	$\sqrt{36} = 6$ because $6 \times 6 = 36$
10. Solutions to $x^2 =$	**Equations** involving **squares** have **two solutions**, one **positive** and one **negative**.	Solve $x^2 = 25$ $x = 5 \text{ or } x = -5$ This can also be written as $x = \pm 5$
11. Cube Number	The number you get when you **multiply a number by itself and itself again**.	1, 8, 27, 64, 125… $2^3 = 2 \times 2 \times 2 = 8$

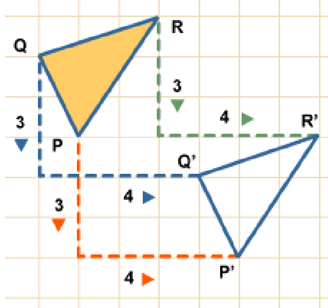
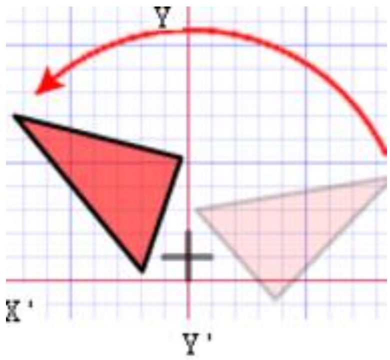
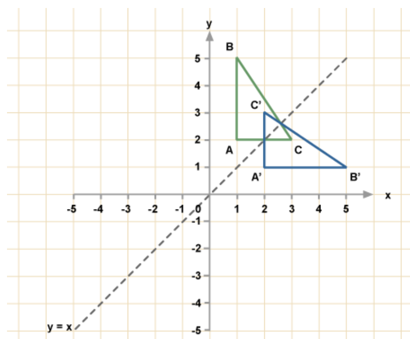
8. Number

12. Cube Root	The **number you multiply by itself and itself again** to get another number. The reverse process of cubing a number.	$\sqrt[3]{125} = 5$ because $5 \times 5 \times 5 = 125$
13. Powers of…	The powers of a number are that **number raised to various powers**.	The powers of 3 are: $3^1 = 3$ $3^2 = 9$ $3^3 = 27$ $3^4 = 81$ etc.
14. Multiplication Index Law	When **multiplying** with the same base (number or letter), **add the powers**. $\boldsymbol{a^m \times a^n = a^{m+n}}$	$7^5 \times 7^3 = 7^8$ $a^{12} \times a = a^{13}$ $4x^5 \times 2x^8 = 8x^{13}$
15. Division Index Law	When **dividing** with the same base (number or letter), **subtract the powers**. $\boldsymbol{a^m \div a^n = a^{m-n}}$	$15^7 \div 15^4 = 15^3$ $x^9 \div x^2 = x^7$ $20a^{11} \div 5a^3 = 4a^8$
16. Brackets Index Laws	When raising a power to another power (with the same base), multiply the powers together. $\boldsymbol{(a^m)^n = a^{mn}}$	$(y^2)^5 = y^{10}$ $(6^3)^4 = 6^{12}$ $(5x^6)^3 = 125x^{18}$
17. Notable Powers	$\boldsymbol{p = p^1}$ $\boldsymbol{p^0 = 1}$ (anything^0 = 1)	$99999^0 = 1$
18. Negative Powers	A negative power performs the reciprocal. $\boldsymbol{a^{-m} = \frac{1}{a^m}}$	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
19. Fractional Powers	The denominator of a fractional power acts as a 'root'. The numerator of a fractional power acts as a normal power. $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$	$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$ $\left(\frac{25}{16}\right)^{\frac{3}{2}} = \left(\frac{\sqrt{25}}{\sqrt{16}}\right)^3 = \left(\frac{5}{4}\right)^3 = \frac{125}{64}$
20. Surd	The **irrational number** that is a **root of a positive integer,** whose value cannot be determined exactly. Surds have **infinite non-recurring decimals**.	$\sqrt{2}$ is a surd because it is a root which cannot be determined exactly. $\sqrt{2} = 1.41421356 \ldots$ which never repeats.
21. Rules of Surds	$\boldsymbol{\sqrt{ab} = \sqrt{a} \times \sqrt{b}}$ $\boldsymbol{\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}}$ $\boldsymbol{a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}}$ $\boldsymbol{\sqrt{a} \times \sqrt{a} = a}$	$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ $\sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$ $2\sqrt{5} + 7\sqrt{5} = 9\sqrt{5}$ $\sqrt{7} \times \sqrt{7} = 7$

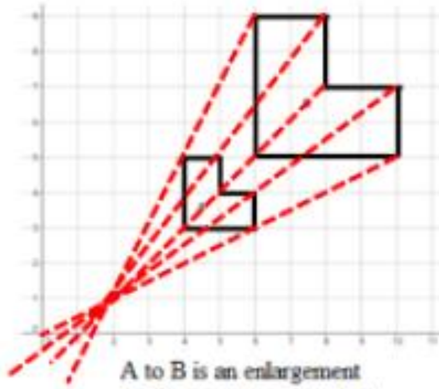
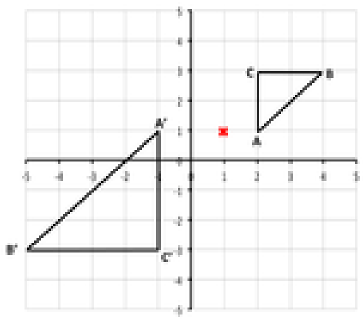
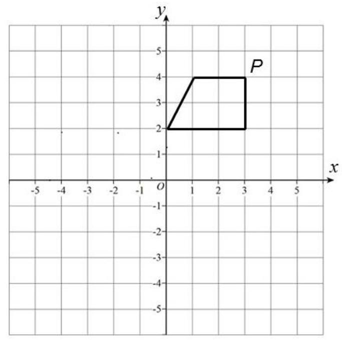
8. Number

22. Rationalise a Denominator	The process of rewriting a fraction so that the **denominator contains only rational numbers**.	$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{6}}{2}$ $\frac{6}{3+\sqrt{7}} = \frac{6(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})}$ $= \frac{18-6\sqrt{7}}{9-7}$ $= \frac{18-6\sqrt{7}}{2} = 9 - 3\sqrt{7}$
23. Standard Form	$\boldsymbol{A \times 10^b}$ $where\ \mathbf{1 \le A < 10}, \quad \boldsymbol{b = integer}$	$8400 = 8.4 \times 10^3$ $0.00036 = 3.6 \times 10^{-4}$
24. Multiplying or Dividing with Standard Form	Multiply: **Multiply the numbers** and **add the powers**. Divide: **Divide the numbers** and **subtract the powers**. Double check your final answer is in correct standard form, adjust if needed.	$(1.2 \times 10^3) \times (4 \times 10^6) = 8.8 \times 10^9$ $(4.5 \times 10^5) \div (3 \times 10^2) = 1.5 \times 10^3$ $(5 \times 10^{-2}) \times (7 \times 10^{-3}) = 35 \times 10^{-5}$ $= 3.5 \times 10^{-4}$
25. Adding or Subtracting with Standard Form	**Convert** in to **ordinary** numbers, **calculate** and then **convert back** in to standard form	$2.7 \times 10^4 + 4.6 \times 10^3$ $= 27000 + 4600 = 31600$ $= 3.16 \times 10^4$

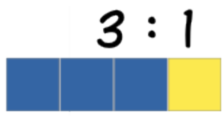
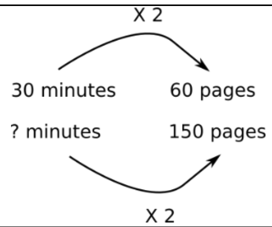
9. Transformations

Topic/Skill	Definition/Tips	Example
1. Translation	**Translate** means to **move a shape**. The shape does not change **size** or **orientation**.	
2. Column Vector	In a column vector, the **top** number moves **left (-) or right (+)** and the **bottom** number moves **up (+) or down (-)**	$\begin{pmatrix}2\\3\end{pmatrix}$ means '2 right, 3 up' $\begin{pmatrix}-1\\-5\end{pmatrix}$ means '1 left, 5 down'
3. Rotation	The size does not change, but the **shape is turned around a point**. Use tracing paper.	Rotate Shape A 90° anti-clockwise about (0,1) 
4. Reflection	The size does not change, but the shape is '**flipped'** like in a **mirror**. Line $\boldsymbol{x} =?$ is a **vertical line**. Line $\boldsymbol{y} =?$ is a **horizontal line**. Line $\boldsymbol{y} = \boldsymbol{x}$ is a **diagonal line**.	Reflect shape C in the line $y = x$ 
5. Enlargement	The shape will get **bigger or smaller**. Multiply each side by the **scale factor**.	Scale Factor = 3 means '3 times larger = multiply by 3' Scale Factor = ½ means 'half the size = divide by 2'

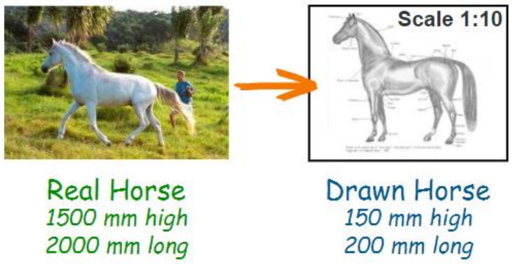
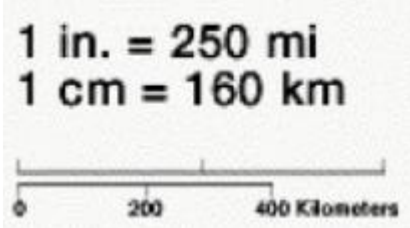
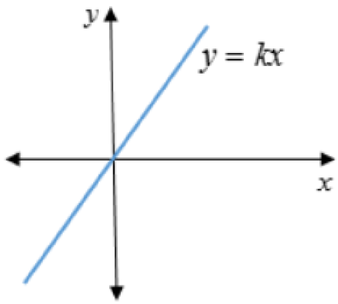
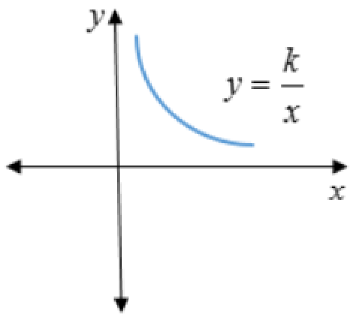
9. Transformations

6. Finding the Centre of Enlargement	Draw **straight lines** through **corresponding corners** of the two shapes. The centre of enlargement is the point **where all the lines cross over**. Be careful with negative enlargements as the corresponding corners will be the other way around (inverted).	 A to B is an enlargement SF 2 about the point (2,1)
7. Describing Transformations	Give the following information when describing each transformation: Look at the number of marks in the question for a hint of how many pieces of information are needed. If you are asked to describe a 'transformation', you need to say the **name of the type of transformation** as well as the other details.	**- Translation, Vector** **- Rotation, Direction, Angle, Centre** **- Reflection, Equation of mirror line** **- Enlargement, Scale factor, Centre of enlargement**
8. Negative Scale Factor Enlargements	Negative enlargements will **look like they have been rotated**. $SF = -2$ will be rotated, and also twice as big.	Enlarge ABC by scale factor -2, centre (1,1) 
9. Invariance	A point, line or shape is invariant if it **does not change/move** when a transformation is performed. An invariant point 'does not vary'.	If shape P is reflected in the $y - axis$, then exactly one vertex is invariant. 

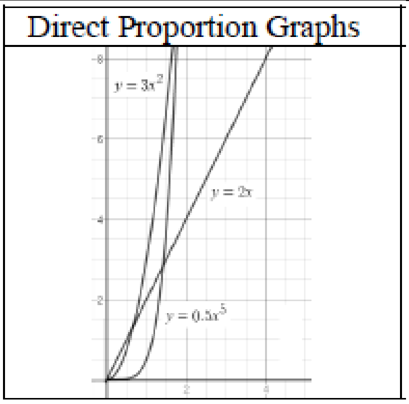
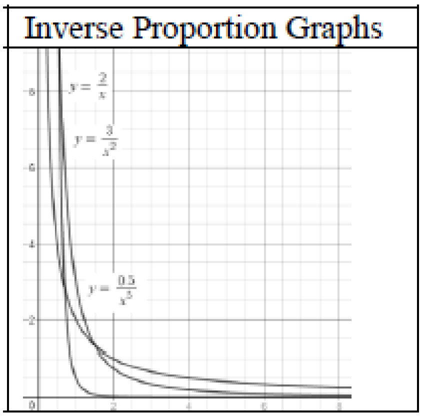
10. Ratio & Proportion

Topic/Skill	Definition/Tips	Example
1. Ratio	Ratio compares the size of **one part** to **another part**. Written using the ':' symbol.	
2. Proportion	Proportion compares the size of **one part** to the size of the **whole**. Usually written as a fraction.	In a class with 13 boys and 9 girls, the proportion of boys is $\frac{13}{22}$ and the proportion of girls is $\frac{9}{22}$
3. Simplifying Ratios	**Divide** all parts of the ratio by a **common factor**.	5 : 10 = 1 : 2 (divide both by 5) 14 : 21 = 2 : 3 (divide both by 7)
4. Ratios in the form 1 : n or n : 1	**Divide** both parts of the ratio by one of the numbers to make **one part equal 1**.	5 : 7 = 1 : $\frac{7}{5}$ in the form 1 : n 5 : 7 = $\frac{5}{7}$: 1 in the form n : 1
5. Sharing in a Ratio	**1. Add** the total parts of the ratio. **2. Divide** the amount to be shared by this value to find the value of one part. **3. Multiply** this value by each part of the ratio. Use only if you **know the total**.	Share £60 in the ratio 3 : 2 : 1. 3 + 2 + 1 = 6 60 ÷ 6 = 10 3 x 10 = 30, 2 x 10 = 20, 1 x 10 = 10 £30 : £20 : £10
6. Proportional Reasoning	Comparing two things using **multiplicative reasoning** and applying this to a new situation. Identify one multiplicative link and use this to find missing quantities.	
7. Unitary Method	Finding the **value of a single unit** and then finding the necessary value by **multiplying** the single unit value.	3 cakes require 450g of sugar to make. Find how much sugar is needed to make 5 cakes. 3 cakes = 450g So 1 cake = 150g (÷ by 3) So 5 cakes = 750 g (x by 5)
8. Ratio already shared	Find what **one part** of the ratio is worth using the **unitary method**.	Money was shared in the ratio 3:2:5 between Ann, Bob and Cat. Given that Bob had £16, found out the total amount of money shared. £16 = 2 parts So £8 = 1 part 3 + 2 + 5 = 10 parts, so 8 x 10 = £80
9. Best Buys	Find the **unit cost** by **dividing** the **price by the quantity**. The **lowest** number is the best value.	8 cakes for £1.28 → 16p each (÷by 8) 13 cakes for £2.05 → 15.8p each (÷by 13) Pack of 13 cakes is best value.

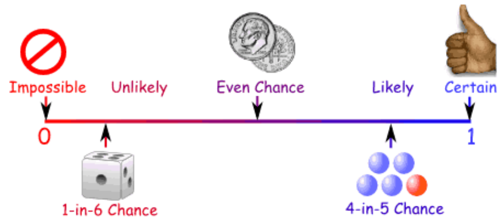
10. Ratio & Proportion

10. Scale	The **ratio** of the **length** in a **model** to the length of the **real** thing.	
11. Scale (Map)	The **ratio** of a **distance on the map** to the actual **distance in real life**.	
12. Direct Proportion	If two quantities are in direct proportion, **as one increases**, the **other increases** by the **same percentage**. If y is directly proportional to x, this can be written as $\boldsymbol{y \propto x}$ An equation of the form $\boldsymbol{y = kx}$ represents direct proportion, where k **is the constant of proportionality**.	
13. Inverse Proportion	If two quantities are inversely proportional, **as one increases**, the **other decreases** by the **same percentage**. If y is inversely proportional to x, this can be written as $\boldsymbol{y \propto \frac{1}{x}}$ An equation of the form $\boldsymbol{y = \frac{k}{x}}$ represents inverse proportion.	
14. Using proportionality formulae	**Direct**: **y = kx** or **y ∝ x** **Inverse**: **y =** $\boldsymbol{\frac{k}{x}}$ or **y ∝** $\boldsymbol{\frac{1}{x}}$ 1. **Solve to find k** using the pair of values in the question. 2. **Rewrite the equation** using the k you have just found. 3. **Substitute the other given value** from the question in to the equation to **find the missing value**.	p is directly proportional to q. When p = 12, q = 4. Find p when q = 20. 1. p = kq 12 = k x 4 so k = 3 2. p = 3q 3. p = 3 x 20 = 60, so p = 60

10. Ratio & Proportion

15. Direct Proportion with powers	Graphs showing **direct proportion** can be written in the form $\boldsymbol{y = kx^n}$ Direct proportion graphs will always start at the origin.	
16. Inverse Proportion with powers	Graphs showing **inverse proportion** can be written in the form $\boldsymbol{y = \frac{k}{x^n}}$ Inverse proportion graphs will never start at the origin.	

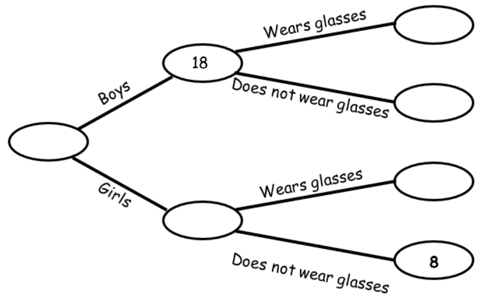
11. Probability

Topic/Skill	Definition/Tips	Example
1. Probability	The **likelihood/chance** of something happening. Is expressed as a number **between 0 (impossible) and 1 (certain)**. Can be expressed as a fraction, decimal, percentage or in words (likely, unlikely, even chance etc.)	
2. Probability Notation	**P(A)** refers to the **probability that event A will occur**.	P(Red Queen) refers to the probability of picking a Red Queen from a pack of cards.
3. Theoretical Probability	$\frac{\textbf{Number of Favourable Outcomes}}{\textbf{Total Number of Possible Outcomes}}$	Probability of rolling a 4 on a fair 6-sided die = $\frac{1}{6}$.
4. Relative Frequency	$\frac{\textbf{Number of Successful Trials}}{\textbf{Total Number of Trials}}$	A coin is flipped 50 times and lands on Tails 29 times. The relative frequency of getting Tails = $\frac{29}{50}$.
5. Expected Outcomes	To find the number of expected outcomes, **multiply** the **probability** by the **number of trials**.	The probability that a football team wins is 0.2 How many games would you expect them to win out of 40? $0.2 \times 40 = 8\ games$
6. Exhaustive	Outcomes are **exhaustive** if they **cover the entire range of possible outcomes**. The **probabilities** of an **exhaustive** set of outcomes **adds up to 1**.	When rolling a six-sided die, the outcomes 1, 2, 3, 4, 5 and 6 are exhaustive, because they cover all the possible outcomes.
7. Mutually Exclusive	Events are mutually exclusive if they **cannot happen at the same time**. The **probabilities** of an exhaustive set of **mutually exclusive** events **adds up to 1**. The probability of something **happening** versus **not happening** is an example of mutually exclusive events.	Examples of mutually exclusive events: - Turning left and right - Heads and Tails on a coin Examples of non mutually exclusive events: - King and Hearts from a deck of cards, because you can pick the King of Hearts

11. Probability

<table>
<tr>
<td>8. Frequency Tree</td>
<td>A diagram showing how information is categorised into various categories.

The numbers at the ends of branches tells us how often something happened (frequency).

The lines connected the numbers are called branches.</td>
<td>
</td>
</tr>
<tr>
<td>9. Sample Space</td>
<td>The set of all possible outcomes of an experiment.</td>
<td>
<table>
<tr><td>+</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
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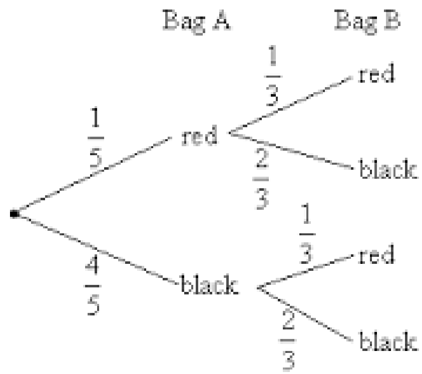
<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
</table>
</td>
</tr>
<tr>
<td>10. Sample</td>
<td>A sample is a small selection of items from a population.

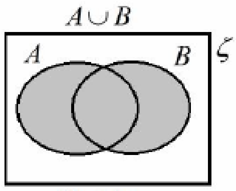
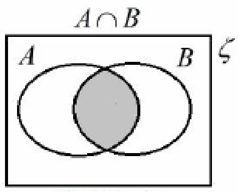
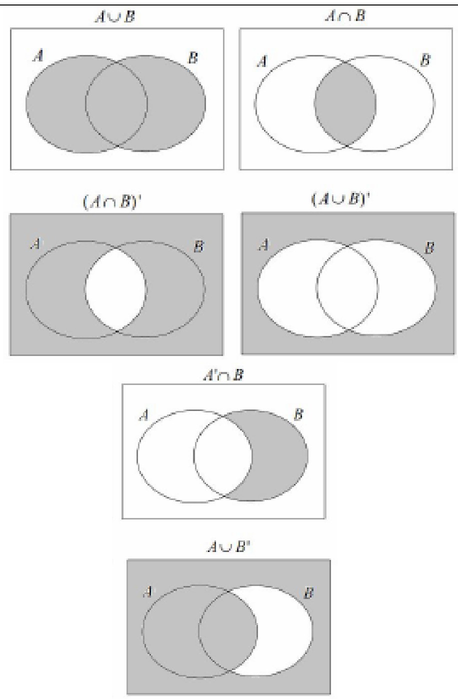
A sample is biased if individuals or groups from the population are not represented in the sample.</td>
<td>A sample could be selecting 10 students from a year group at school.</td>
</tr>
<tr>
<td>11. Sample Size</td>
<td>The larger a sample size, the closer those probabilities will be to the true probability.</td>
<td>A sample size of 100 gives a more reliable result than a sample size of 10.</td>
</tr>
<tr>
<td>12. Tree Diagrams</td>
<td>Tree diagrams show all the possible outcomes of an event and calculate their probabilities.

All branches must add up to 1 when adding downwards.
This is because the probability of something not happening is 1 minus the probability that it does happen.

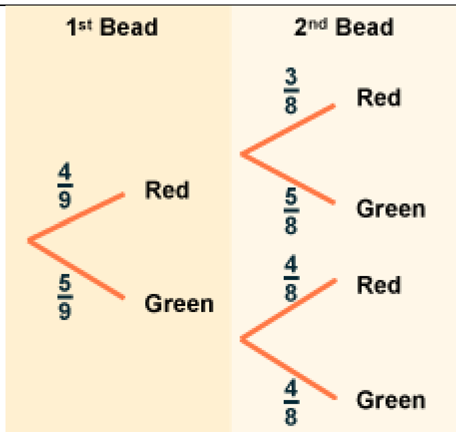
Multiply going across a tree diagram.

Add going down a tree diagram.</td>
<td>
</td>
</tr>
<tr>
<td>13. Independent Events</td>
<td>The outcome of a previous event does not influence/affect the outcome of a second event.</td>
<td>An example of independent events could be <u>replacing</u> a counter in a bag after picking it.</td>
</tr>
</table>

11. Probability

14. Dependent Events	The outcome of a **previous event does influence/affect the outcome of a second event**.	An example of dependent events could be not replacing a counter in a bag after picking it. '<u>Without replacement</u>'
15. Probability Notation	**P(A)** refers to the **probability that event A will occur**. **P(A')** refers to the **probability that event A will <u>not</u> occur**. **P(A ∪ B)** refers to the **probability that event A <u>or</u> B <u>or</u> both will occur.** **P(A ∩ B)** refers to the **probability that <u>both</u> events A and B will occur (at the same time).**	P(Red Queen) refers to the probability of picking a Red Queen from a pack of cards. P(Blue') refers to the probability that you do not pick Blue. P(Blonde ∪ Right Handed) refers to the probability that you pick someone who is Blonde or Right Handed or both. P(Blonde ∩ Right Handed) refers to the probability that you pick someone who is both Blonde and Right Handed.
16. Venn Diagrams	A Venn Diagram shows the **relationship between a group of different things** and how they overlap. You may be asked to shade Venn Diagrams as shown below and to the right.  The Union 'A or B or Both'  The Intersection 'A and B'	
17. Venn Diagram Notation	∈ means '**element of a set**' (a value in the set) { } means the collection of values in the set. ξ means the '**universal set**' (all the values to consider in the question) **A'** means **'not in set A'** (called **complement**) **A ∪ B** means **'A or B or both'** (called **Union**) **A ∩ B** means **'A and B** (called **Intersection**)	Set A is the even numbers less than 10. A = {2, 4, 6, 8} Set B is the prime numbers less than 10. B = {2, 3, 5, 7} A ∪ B = {2, 3, 4, 5, 6, 7, 8} A ∩ B = {2}

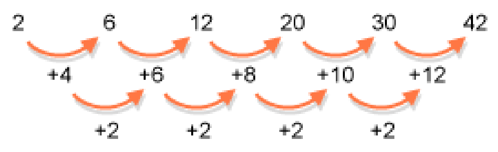
11. Probability

18. AND rule for Probability	When two events, A and B, are **independent**: $$\boldsymbol{P(A \text{ and } B) = P(A) \times P(B)}$$	What is the probability of rolling a 4 and flipping a Tails? $$P(4 \text{ and } Tails) = P(4) \times P(Tails)$$ $$= \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$
19. OR rule for Probability	When two events, A and B, are **mutually exclusive**: $$\boldsymbol{P(A \text{ or } B) = P(A) + P(B)}$$	What is the probability of rolling a 2 or rolling a 5? $$P(2 \text{ or } 5) = P(2) + P(5)$$ $$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$
20. Conditional Probability	The probability of an event A happening, **given that** event B has already happened. With conditional probability, check if the numbers on the second branches of a tree diagram changes. For example, if you have 4 red beads in a bag of 9 beads and pick a red bead on the first pick, then there will be 3 red beads left out of 8 beads on the second pick.	
21. Combination	A collection of things, where the **order does not matter**.	How many combinations of two ingredients can you make with apple, banana and cherry? Apple, Banana Apple, Cherry Banana, Cherry 3 combinations
22. Permutation	A collection of things, where the **order does matter**.	You want to visit the homes of three friends, Alex (A), Betty (B) and Chandra (C) but haven't decided the order. What choices do you have? ABC ACB BAC BCA CAB CBA

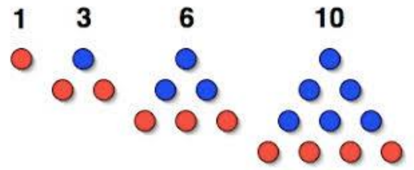
11. Probability

23. Permutations with Repetition	When something has n different types, there are $\boldsymbol{n}$ **choices each time**. Choosing r of something that has n different types, the permutations are: $n \times n \times \ldots (r\ times) = \boldsymbol{n^r}$	How many permutations are there for a three-number combination lock? 10 numbers to choose from $\{1, 2, \ldots .10\}$ and we choose 3 of them → $10 \times 10 \times 10 = 10^3 = 1000$ permutations.
24. Permutations without Repetition	We have to **reduce the number of available choices each time**. One you have chosen something, you cannot choose it again.	How many ways can you order 4 numbered balls? $4 \times 3 \times 2 \times 1 = 24$
25. Factorial	The factorial symbol '!' means to multiply a series of descending integers to 1. Note: $0! = 1$	$4! = 4 \times 3 \times 2 \times 1 = 24$
26. Product Rule for Counting	If there are $\boldsymbol{x}$ **ways of doing something** and $\boldsymbol{y}$ **ways of doing something else**, then there are $\boldsymbol{xy}$ **ways of performing both**.	To choose one of $\{A, B, C\}$ and one of $\{X, Y\}$ means to choose one of $\{AX, AY, BX, BY, CX, CY\}$ The rule says that there are $3 \times 2 = 6$ choices.

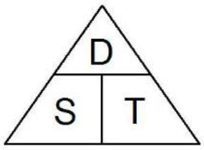
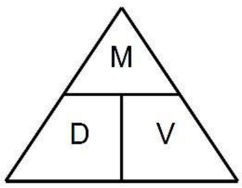
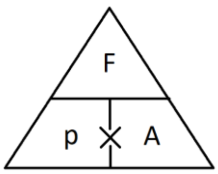
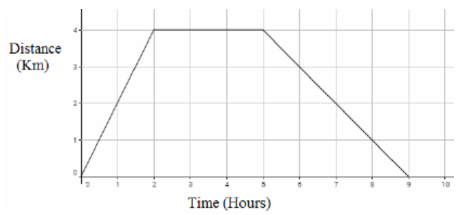
12. Formula & Sequences

Topic/Skill	Definition/Tips	Example
1. Linear Sequence	A number pattern with a **common difference**.	2, 5, 8, 11… is a linear sequence
2. Term	**Each value** in a sequence is called a term.	In the sequence 2, 5, 8, 11…, 8 is the third term of the sequence.
3. Term-to-term rule	A rule which allows you to **find the next term** in a sequence if you **know the previous term**.	First term is 2. Term-to-term rule is 'add 3' Sequence is: 2, 5, 8, 11…
4. nth term	A rule which allows you to **calculate the term** that is in the **nth position** of the sequence. Also known as the 'position-to-term' rule. **n** refers to the **position** of a term in a sequence.	nth term is $3n - 1$ The 100th term is $3 \times 100 - 1 = 299$
5. Finding the nth term of a linear sequence	1. Find the **difference**. 2. **Multiply that by n.** 3. Substitute $n = 1$ to **find out what number you need to add or subtract to get the first number in the sequence**.	Find the nth term of: 3, 7, 11, 15… 1. Difference is +4 2. Start with $4n$ 3. $4 \times 1 = 4$, so we need to subtract 1 to get 3. nth term = $4n - 1$
6. Fibonacci type sequences	A sequence where the next number is found by **adding up the previous two terms**	The Fibonacci sequence is: 1,1,2,3,5,8,13,21,34 ... An example of a Fibonacci-type sequence is: $4, 7, 11, 18, 29$ ...
7. Geometric Sequence	A sequence of numbers where each term is found by **multiplying the previous one** by a number called the **common ratio, r**.	An example of a geometric sequence is: $2, 10, 50, 250$ ... The common ratio is 5 Another example of a geometric sequence is: $81, -27, 9, -3, 1$ ... The common ratio is $-\frac{1}{3}$
8. Quadratic Sequence	A sequence of numbers where the **second difference is constant**. A quadratic sequence will have a n^2 term.	

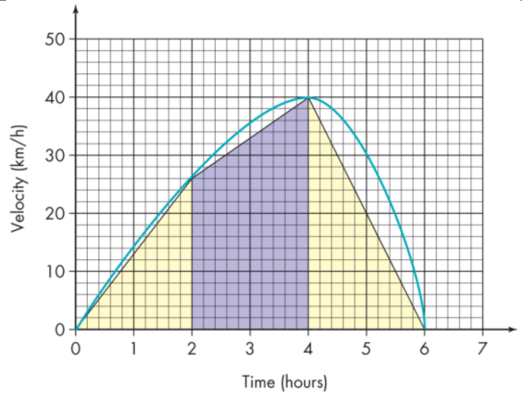
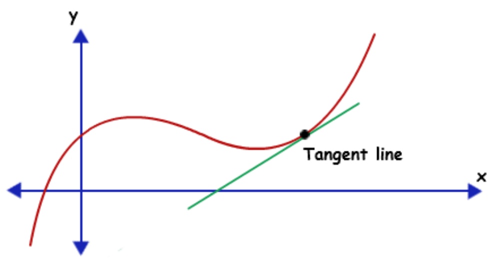
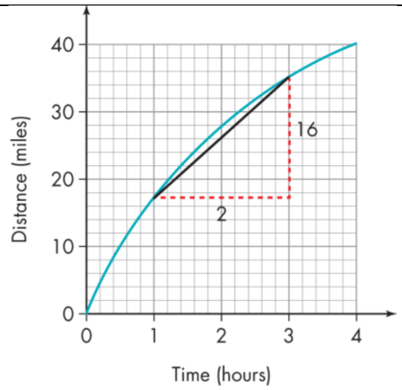
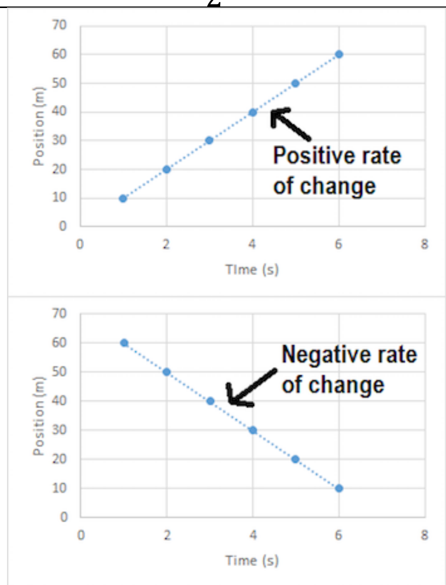
12. Formula & Sequences

9. nth term of a geometric sequence	ar^{n-1} where a is the first term and r is the common ratio	The nth term of $2, 10, 50, 250$ .... Is $2 \times 5^{n-1}$
10. nth term of a quadratic sequence	1. Find the first and second differences. 2. Halve the second difference and multiply this by n^2. 3. Substitute $n = 1,2,3,4$ ... into your expression so far. 4. Subtract this set of numbers from the corresponding terms in the sequence from the question. 5. Find the nth term of this set of numbers. 6. Combine the nth terms to find the overall nth term of the quadratic sequence. Substitute values in to check your nth term works for the sequence.	Find the nth term of: 4, 7, 14, 25, 40.. Answer: Second difference = +4 → nth term = $2n^2$ Sequence: 4, 7, 14, 25, 40 $2n^2$ 2, 8, 18, 32, 50 Difference: 2, -1, -4, -7, -10 Nth term of this set of numbers is $-3n + 5$ Overall nth term: $2n^2 - 3n + 5$
	Alternative Method: 1. Find the first and second differences. 2. Use the following equations and set them equal to the first number in each row. a+b+c 3a+b 2a Then solve from the bottom up. Now write your answer as an^2+bn+c	Sequence: **4**, 7, 14, 25, 40 1st diff: **3**, 7, 11, 15 2nd diff: **4**, 4, 4 2a=**4**, so **a=2** 3a+b=**3** 3(2)+b=3 so **b=-3** a+b+c=**4** (2)+(-3)+c=4 so **c=5** Nth term = $\mathbf{2}n^2 - \mathbf{3}n + \mathbf{5}$
11. Triangular numbers	The sequence which comes from a pattern of dots that form a triangle. $1, 3, 6, 10, 15, 21$ ...	
12. Metric System	A system of measures based on: - the metre for length - the kilogram for mass - the second for time **Length: mm, cm, m, km** **Mass: mg, g, kg** **Volume: ml, cl, l**	$1 kilometres = 1000\ metres$ $1\ metre = 100\ centimetres$ $1\ centimetre = 10\ millimetres$ $1\ kilogram = 1000\ grams$
13. Imperial System	A system of weights and measures originally developed in England, usually based on human quantities **Length: inch, foot, yard, miles** **Mass: lb, ounce, stone** **Volume: pint, gallon**	$1 lb = 16\ ounces$ $1\ foot = 12\ inches$ $1\ gallon = 8\ pints$

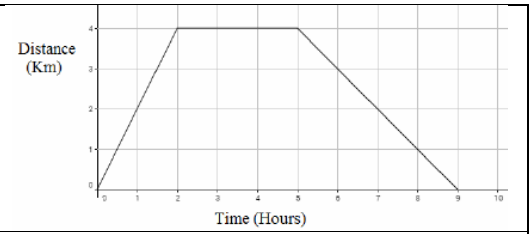
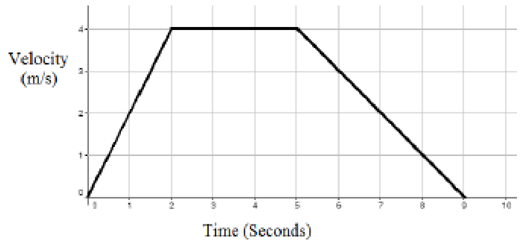
12. Formula & Sequences

14. Metric and Imperial Units	Use the **unitary method** to convert between metric and imperial units.	$5\ miles \approx 8\ kilometres$ $1\ gallon \approx 4.5\ litres$ $2.2\ pounds \approx 1\ kilogram$ $1\ inch = 2.5\ centimetres$
15. Speed, Distance, Time	**Speed = Distance ÷ Time** **Distance = Speed x Time** **Time = Distance ÷ Speed**  Remember the correct units.	Speed = 4mph Time = 2 hours Find the Distance. $D = S \times T = 4 \times 2 = 8\ miles$
16. Density, Mass, Volume	**Density = Mass ÷ Volume** **Mass = Density x Volume** **Volume = Mass ÷ Density**  Remember the correct units.	Density = 8kg/m^3 Mass = 2000g Find the Volume. $V = M \div D = 2 \div 8 = 0.25m^3$
17. Pressure, Force, Area	**Pressure = Force ÷ Area** **Force = Pressure x Area** **Area = Force ÷ Pressure**  Remember the correct units.	Pressure = 10 Pascals Area = 6cm^2 Find the Force $F = P \times A = 10 \times 6 = 60\ N$
18. Distance-Time Graphs	You can find the **speed** from the **gradient** of the line (Distance ÷ Time) The steeper the line, the quicker the speed. A **horizontal** line means the object is not moving (**stationary**).	

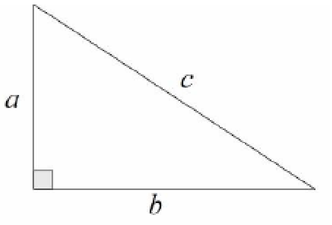
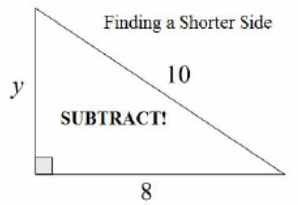
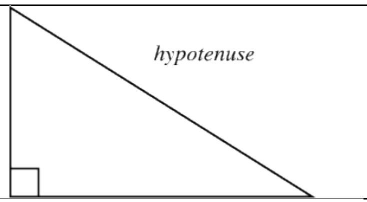
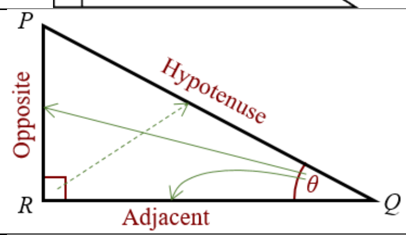
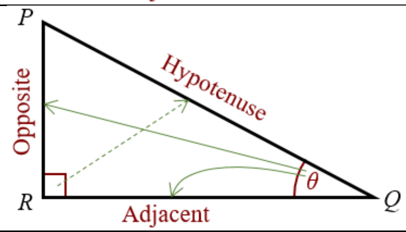
12. Formula & Sequences

19. Area Under a Curve	To find the area under a curve, **split it up into simpler shapes** – such as rectangles, triangles and trapeziums – that approximate the area.	
20. Tangent to a Curve	A straight **line** that **touches** a curve at **exactly one point**.	
21. Gradient of a Curve	The **gradient of a curve** at a point is the same as the **gradient of the tangent** at that point. 1. Draw a tangent carefully at the point. 2. Make a right-angled triangle. 3. Use the measurements on the axes to calculate the rise and run (change in y and change in x) 4. Calculate the gradient.	 $Gradient = \frac{Change\ in\ y}{Change\ in\ x}$ $= \frac{16}{2} = 8$
22. Rate of Change	The rate of change at a particular instant in time is represented by the **gradient of the tangent to the curve** at that point.	

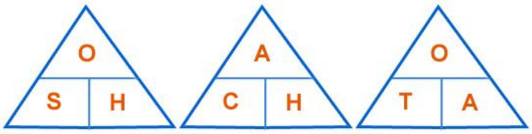
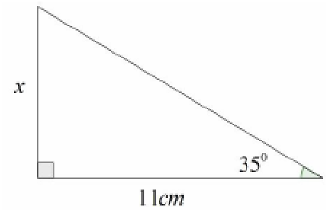
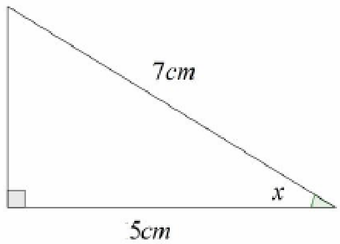
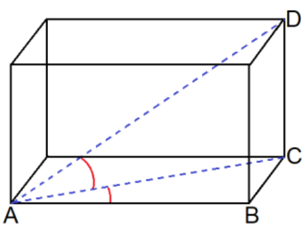
12. Formula & Sequences

23. Distance-Time Graphs	You can find the **speed** from the **gradient** of the line (Distance ÷ Time) The steeper the line, the quicker the speed. A **horizontal** line means the object is not moving (**stationary**).	
24. Velocity-Time Graphs	You can find the **acceleration** from the **gradient** of the line (Change in Velocity ÷ Time) The steeper the line, the quicker the acceleration. A **horizontal line** represents no acceleration, meaning a **constant velocity**. The **area** under the graph is the **distance.**	

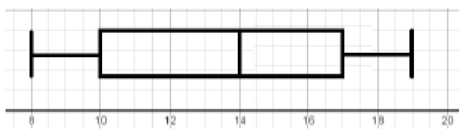
13. Pythagoras & Trigonometry

Topic/Skill	Definition/Tips	Example
1. Pythagoras' Theorem	For any **right-angled triangle**: $\boldsymbol{a^2 + b^2 = c^2}$  Used to find **missing lengths**. a and b are the shorter sides, c is the **hypotenuse** (**longest side**).	 $a = y,\ b = 8,\ c = 10$ $a^2 = c^2 - b^2$ $y^2 = 100 - 64$ $y^2 = 36$ $y = 6$
2. 3D Pythagoras' Theorem	Find missing lengths by **identifying right angled triangles**. You will often have to find a missing length you are not asked for before finding the missing length you are asked for.	Can a pencil that is 20cm long fit in a pencil tin with dimensions 12cm, 13cm and 9cm? The pencil tin is in the shape of a cuboid. Hypotenuse of the base = $\sqrt{12^2 + 13^2} = 17.7$ Diagonal of cuboid = $\sqrt{17.7^2 + 9^2} = 19.8cm$ No, the pencil cannot fit.
1. Trigonometry	The **study** of **triangles**. In particular, the relationship between side lengths and angles of triangles.	
2. Hypotenuse	The **longest side** of a **right-angled triangle**. Is always **opposite** the **right angle**.	
3. Adjacent	The side **next to** the angle involved in the question.	
4. Opposite	The side **opposite** the angle involved in the question.	

13. Pythagoras & Trigonometry

5. Trigonometric Formulae	Use **SOHCAHTOA**. $\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{A}$  When finding a missing angle, use the ‘inverse’ trigonometric function by pressing the ‘shift’ button on the calculator.	 Use ‘Opposite’ and ‘Adjacent’, so use ‘tan’ $\tan 35 = \frac{x}{11}$ $x = 11\tan 35 = 7.70cm$  Use ‘Adjacent’ and ‘Hypotenuse’, so use ‘cos’ $\cos x = \frac{5}{7}$ $x = cos^{-1}\left(\frac{5}{7}\right) = 44.4°$
6. 3D Trigonometry	Find missing lengths by **identifying right angled triangles**. You will often have to find a missing length you are not asked for before finding the missing length you are asked for. You may have to use Pythagoras too.	

14. Handling Data 2

Topic/Skill	Definition/Tips	Example
1. Lower Quartile	**Divides** the **bottom half** of the data into **two halves**. **LQ =** $\boldsymbol{Q_1 = \frac{(n+1)}{4} th}$ **value**	Find the lower quartile of: 2, **3**, 4, 5, 6, 6, 7 $Q_1 = \frac{(7+1)}{4} = 2nd$ value → 3
2. Upper Quartile	**Divides** the **top half** of the data into **two halves**. **UQ =** $\boldsymbol{Q_3 = \frac{3(n+1)}{4} th}$ **value**	Find the upper quartile of: 2, 3, 4, 5, 6, **6**, 7 $Q_3 = \frac{3(7+1)}{4} = 6th$ value → 6
3. Interquartile Range	The **difference** between the **upper quartile and lower quartile.** $\boldsymbol{IQR = Q_3 - Q_1}$ The **smaller** the **interquartile range**, the **more consistent** the data or the **larger** the interquartile range, the **more variable** the data.	Find the IQR of: 2, 3, 4, 5, 6, 6, 7 $IQR = Q_3 - Q_1 = 6 - 3 = 3$
4. Box Plots	The **minimum**, **lower quartile**, **median**, **upper quartile** and **maximum** are shown on a box plot. A box plot can be drawn independently or from a cumulative frequency diagram. Each section represents a **quarter** of the data.	Students sit a maths test. The highest score is 19, the lowest score is 8, the median is 14, the lower quartile is 10 and the upper quartile is 17. Draw a box plot to represent this information. 
5. Comparing Box Plots	Write two sentences. 1. Compare the **averages** using the **medians** for two sets of data. 2. Compare the **spread** of the data using the **range or IQR** for two sets of data. The smaller the range/IQR, the more consistent the data. You must compare box plots **in the context of the problem**.	'On average, students in class A were more successful on the test than class B because their median score was higher.' 'Students in class B were more consistent than class A in their test scores as their IQR was smaller.'

14. Handling Data 2

6. Histograms

A visual way to display frequency data using bars.

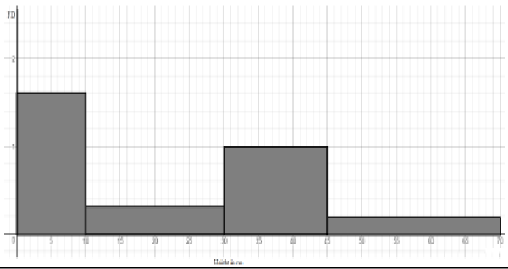
Bars can be **unequal in width**.

Histograms show **frequency density** on the **y-axis**, not frequency.

$$\textbf{Frequency Density} = \frac{\textbf{Frequency}}{\textbf{Class Width}}$$

Height(cm)	Frequency
$0 < h \leq 10$	8
$10 < h \leq 30$	6
$30 < h \leq 45$	15
$45 < h \leq 70$	5

Frequency Density (FD)
$8 \div 5 = 1.6$
$6 \div 20 = 0.3$
$15 \div 15 = 1$
$5 \div 25 = 0.2$

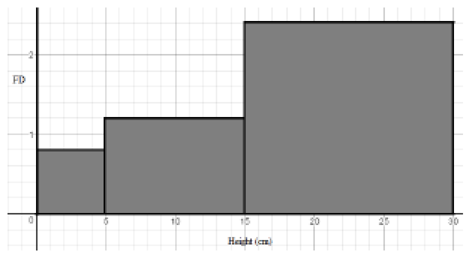


7. Interpreting Histograms

The **area** of the bar is proportional to the **frequency** of that class interval.

$$\textbf{Frequency} = \textbf{Freq Density} \times \textbf{Class Width}$$

A histogram shows information about the heights of a number of plants. 4 plants were less than 5cm tall. Find the number of plants more than 5cm tall.



Above 5cm:
1.2 x 10 + 2.4 x 15 = 12 + 36 = 48

8. Cumulative Frequency

Cumulative Frequency is a **running total**.

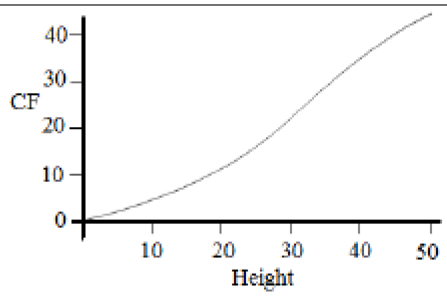
Age	Frequency
$0 < a \leq 10$	15
$10 < a \leq 40$	35
$40 < a \leq 50$	10

Cumulative Frequency
15
$15 + 35 = 50$
$50 + 10 = 60$

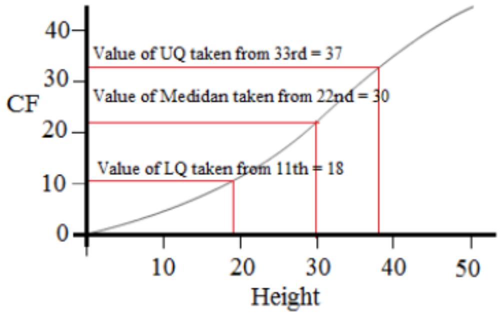
9. Cumulative Frequency Diagram

A cumulative frequency diagram is a **curve that goes up**. It looks a little like a stretched-out **S shape**.

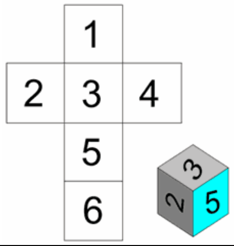
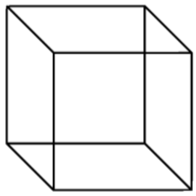
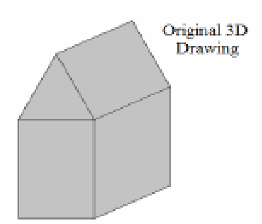
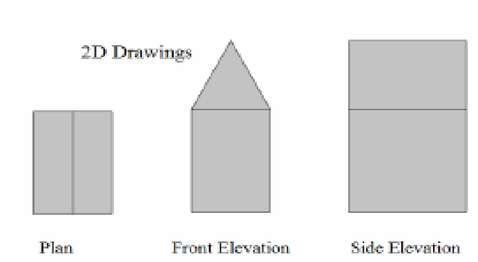
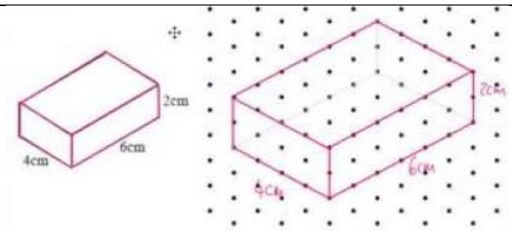
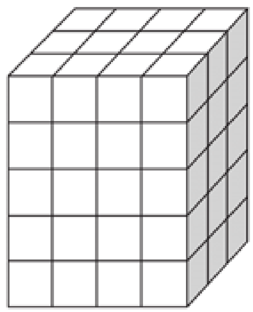
Plot the cumulative frequencies at the **end-point** of each interval.



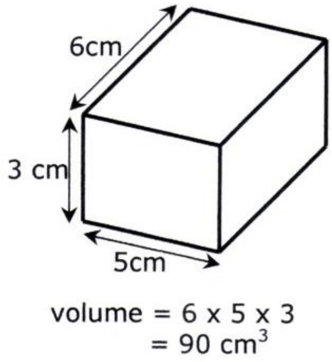
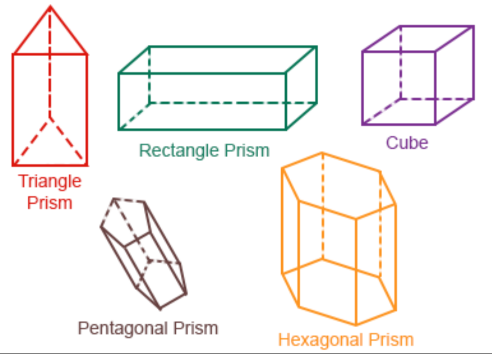
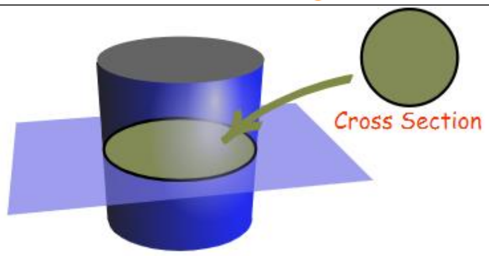
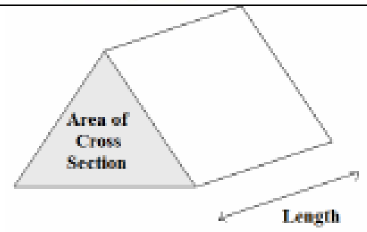
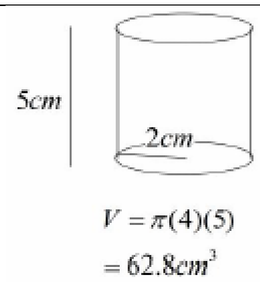
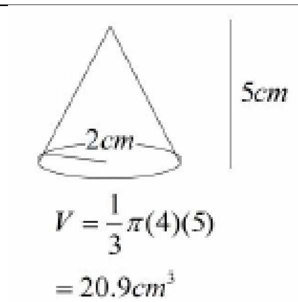
14. Handling Data 2

10. Quartiles from Cumulative Frequency Diagram	**Lower Quartile** (Q1): **25%** of the data is less than the lower quartile. **Median** (Q2): **50%** of the data is less than the median. **Upper Quartile** (Q3): **75%** of the data is less than the upper quartile. **Interquartile Range (IQR)**: represents the **middle 50%** of the data.	 $IQR = 37 - 18 = 19$
11. Hypothesis	**A statement that might be true, which can be tested.**	Hypothesis: 'Large dogs are better at catching tennis balls than small dogs'. We can test this hypothesis by having hundreds of different sized dogs try to catch tennis balls.

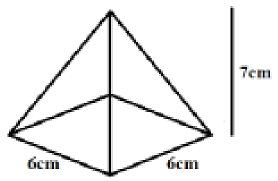
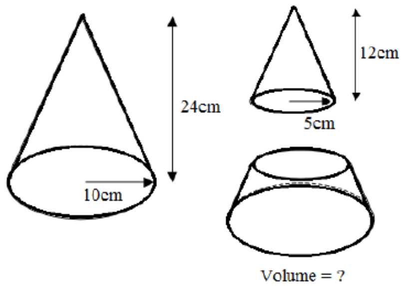
15. Volume

Topic/Skill	Definition/Tips	Example
1. Net	A pattern that you can **cut and fold** to make a **model** of a **3D shape**.	
2. Properties of Solids	**Faces = flat surfaces** **Edges = sides/lengths** **Vertices = corners**	A cube has 6 faces, 12 edges and 8 vertices. 
3. Plans and Elevations	This takes 3D drawings and produces 2D drawings. **Plan View**: from **above** **Side Elevation**: from the **side** **Front Elevation**: from the **front**	 
4. Isometric Drawing	A method for visually **representing 3D objects in 2D**.	
5. Volume	Volume is a measure of the amount of space inside a solid shape. Units: mm^3, cm^3, m^3 etc.	 Vol = 60units^3

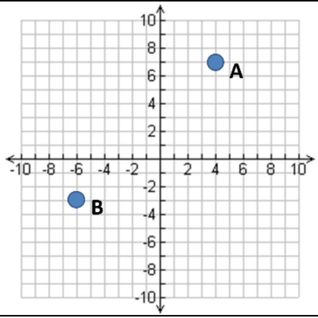
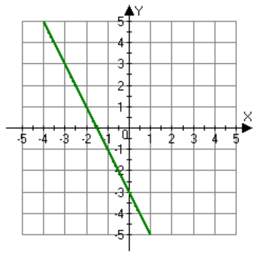
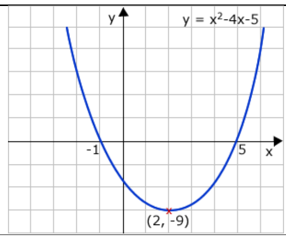
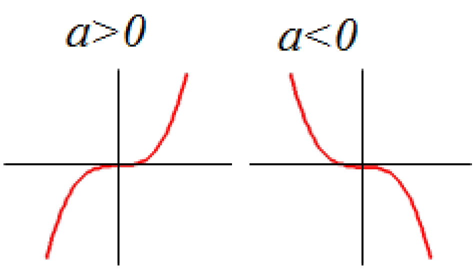
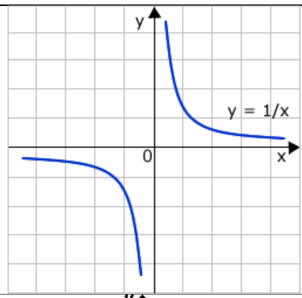
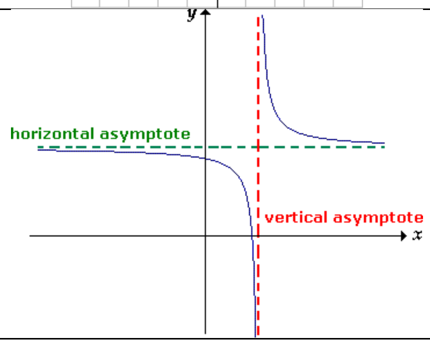
15. Volume

6. Volume of a Cube/Cuboid	$V = Length \times Width \times Height$ $V = L \times W \times H$ You can also use the Volume of a Prism formula for a cube/cuboid.	
7. Prism	A prism is a 3D shape whose **cross section is the same** throughout.	
8. Cross Section	The **cross section** is the **shape** that **continues** all the way **through the prism**.	
9. Volume of a Prism	$V = Area\ of\ Cross\ Section \times Length$ $V = A \times L$	
10. Volume of a Cylinder	$V = \pi r^2 h$	
11. Volume of a Cone	$V = \frac{1}{3}\pi r^2 h$	

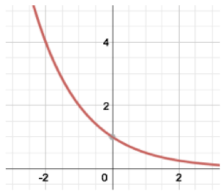
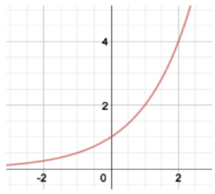
15. Volume

12. Volume of a Pyramid	$$\boldsymbol{Volume = \frac{1}{3}Bh}$$ where B = area of the base	 $V = \frac{1}{3} \times 6 \times 6 \times 7 = 84cm^3$
13. Volume of a Sphere	$$\boldsymbol{V = \frac{4}{3}\pi r^3}$$ Look out for hemispheres – just halve the volume of a sphere.	Find the volume of a sphere with diameter 10cm. $V = \frac{4}{3}\pi(5)^3 = \frac{500\pi}{3}cm^3$
14. Frustums	A frustum is a solid (usually a cone or pyramid) with the **top removed**. Find the volume of the whole shape, then take away the volume of the small cone/pyramid removed at the top. The whole shape and the part removed will be mathematically similar shapes. You might need to establish a scale factor from this fact.	 $V = \frac{1}{3}\pi(10)^2(24) - \frac{1}{3}\pi(5)^2(12)$ $= 700\pi cm^3$

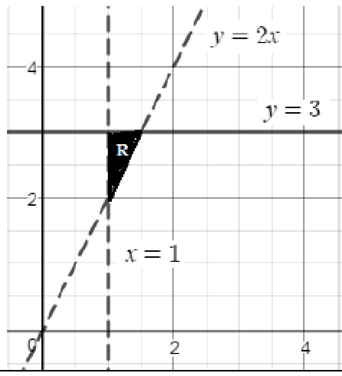
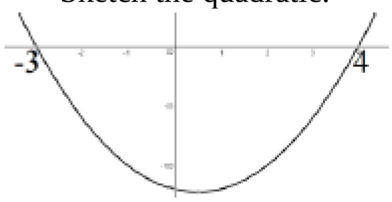
16. Algebraic Graphs

Topic/Skill	Definition/Tips	Example
1. Coordinates	Written in **pairs**. The **first** term is the **x-coordinate** (movement **across**). The **second** term is the **y-coordinate** (movement **up or down**)	 A: (4,7) B: (-6,-3)
2. Linear Graph	**Straight line** graph. The **equation** of a linear graph can contain an **x-term**, a **y-term** and a **number**.	Example:  Other examples: $x = y$ $y = 4$ $x = -2$ $y = 2x - 7$ $y + x = 10$ $2y - 4x = 12$
3. Quadratic Graph	A '**U-shaped**' curve called a **parabola**. The equation is of the form $y = \boldsymbol{ax^2 + bx + c}$, where a, b and c are numbers, $\boldsymbol{a \neq 0}$. If $\boldsymbol{a < 0}$**,** the parabola is **upside down**.	
4. Cubic Graph	The equation is of the form $\boldsymbol{y = ax^3 + k}$, where $\boldsymbol{k}$ **is an number**. If $\boldsymbol{a > 0}$, the curve is **increasing**. If $\boldsymbol{a < 0}$, the curve is **decreasing**.	
5. Reciprocal Graph	The equation is of the form $\boldsymbol{y = \frac{A}{x}}$, where $\boldsymbol{A}$ **is a number** and $\boldsymbol{x \neq 0}$. The graph has **asymptotes** on the **x-axis and y-axis**.	
6. Asymptote	A **straight line** that a graph **approaches** but **never touches**.	

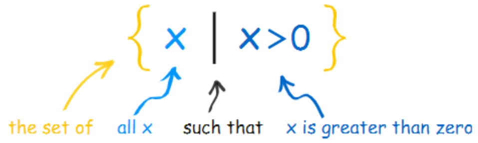
16. Algebraic Graphs

7. Exponential Graph	The equation is of the form $\boldsymbol{y = a^x}$, where $\boldsymbol{a}$ is a number called the **base**. If $\boldsymbol{a > 1}$ the graph **increases**. If $\boldsymbol{0 < a < 1}$, the graph **decreases**. The graph has an **asymptote** which is the **x-axis**. The **y-intercept** of the graph $y = a^x$ is $\boldsymbol{(0, 1)}$.	 

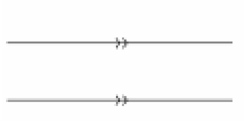
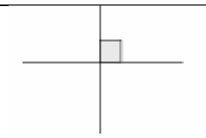
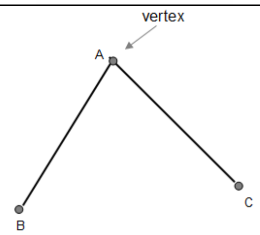
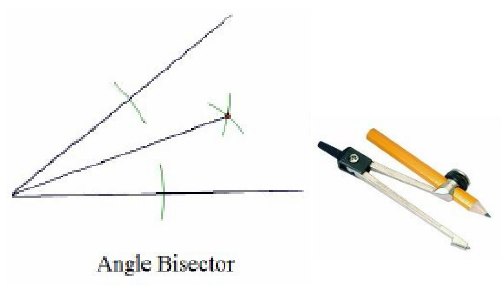
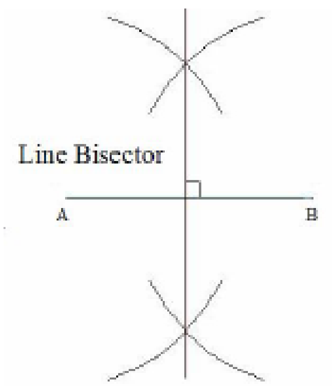
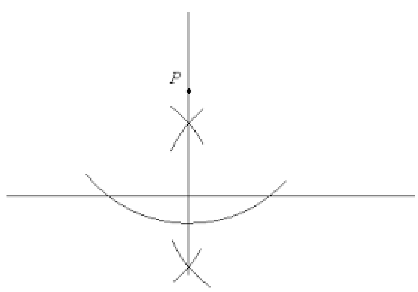
17. Inequalities

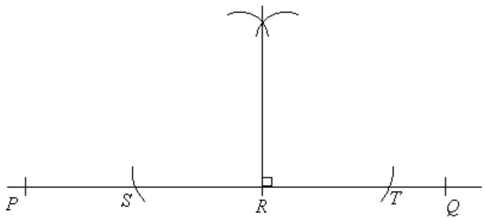
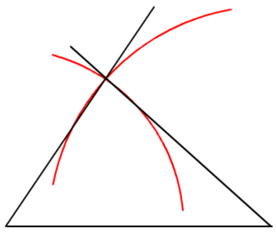
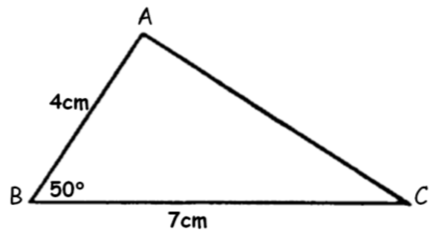
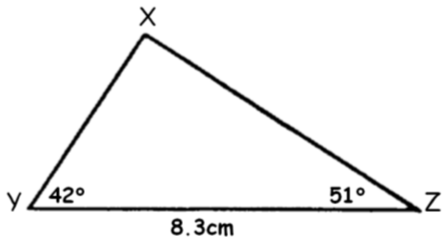
Topic/Skill	Definition/Tips	Example
1. Inequality	An inequality says that two values are **not equal**. $a \neq b$ means that a is not equal to b.	$7 \neq 3$ $x \neq 0$
2. Inequality symbols	$\boldsymbol{x > 2}$ means **x is greater than 2** $\boldsymbol{x < 3}$ means **x is less than 3** $\boldsymbol{x \geq 1}$ means **x is greater than or equal to 1** $\boldsymbol{x \leq 6}$ means **x is less than or equal to 6**	State the integers that satisfy $-2 < x \leq 4$. -1, 0, 1, 2, 3, 4
3. Inequalities on a Number Line	Inequalities can be shown on a number line. **Open circles** are used for numbers that are **less than or greater than** ($<$ *or* $>$) **Closed circles** are used for numbers that are **less than or equal or greater than or equal** ($\leq$ *or* $\geq$)	 $x \geq 0$  $x < 2$  $-5 \leq x < 4$
4. Graphical Inequalities	Inequalities can be represented on a coordinate grid. If the inequality is **strict** ($x > 2$) then use a **dotted line**. If the inequality is **not strict** ($x \leq 6$) then use a **solid line**. **Shade** the **region** which satisfies all the inequalities.	Shade the region that satisfies: $y > 2x, x > 1$ *and* $y \leq 3$ 
5. Quadratic Inequalities	**Sketch the quadratic graph** of the inequality. If the expression is $>$ ***or*** $\geq$ then the answer will be **above the x-axis**. If the expression is $<$ ***or*** $\leq$ then the answer will be **below the x-axis**. Look carefully at the inequality symbol in the question. Look carefully if the quadratic is a **positive or negative parabola**.	Solve the inequality $x^2 - x - 12 < 0$ Sketch the quadratic:  The required region is below the x-axis, so the final answer is: $-3 < x < 4$ If the question had been > 0, the answer would have been: $x < -3$ *or* $x > 4$

17. Inequalities

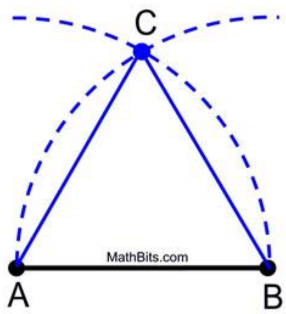
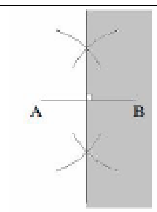
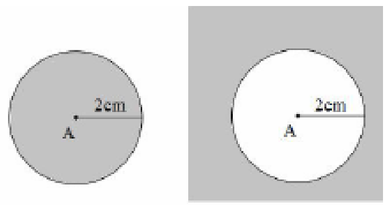
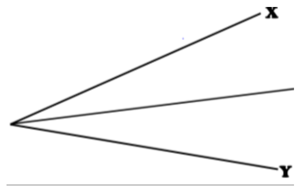
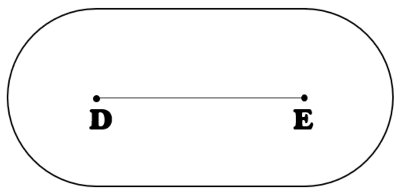
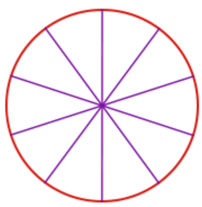
6. Set Notation	A **set** is a **collection of things**, usually numbers, denoted with brackets { } $\{x \mid x \geq 7\}$ means 'the set of all x's, such that x is greater than or equal to 7' The 'x' can be replaced by any letter. Some people use ':' instead of '|'	$\{3, 6, 9\}$ is a set.  $\{x : -2 \leq x < 5\}$

18. Construction & Congruence

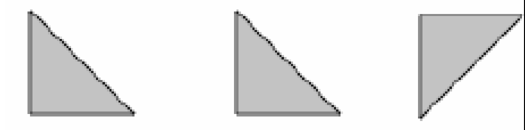
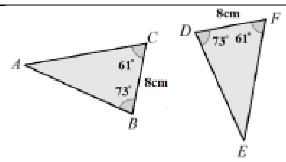
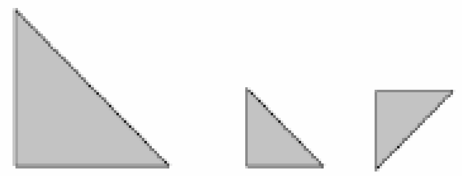
Topic/Skill	Definition/Tips	Example
1. Parallel	Parallel lines never meet.	
2. Perpendicular	Perpendicular lines are at right angles. There is a 90° angle between them.	
3. Vertex	A corner or a point where two lines meet.	
4. Angle Bisector	**Angle Bisector: Cuts the angle in half**. 1. Place the sharp end of a pair of compasses on the vertex. 2. Draw an arc, marking a cut on each line. 3. Without changing the compass put the compass on each 'cut' point and mark a centre point where two arcs cross over. 4. Use a ruler to draw a line through the vertex and centre point.	
5. Perpendicular Bisector	**Perpendicular Bisector: Cuts a line in half and at right angles**. 1. Put the sharp point of a pair of compasses on A. 2. Open the compass over half way on the line. 3. Draw an arc above and below the line. 4. Without changing the compass, repeat from point B. 5. Draw a straight line through the two intersecting arcs.	
6. Perpendicular from an External Point	The **perpendicular distance** from a point to a line is the **shortest distance** to that line. 1. Put the sharp point of a pair of compasses on the point. 2. Draw an arc that crosses the line twice. 3. Place the sharp point of the compass on one of these points, open over half way and draw an arc above and below the line. 4. Repeat from the other point on the line. 5. Draw a straight line through the two intersecting arcs.	

7. Perpendicular from a Point on a Line	Given line PQ and point R on the line: 1. Put the sharp point of a pair of compasses on point R. 2. Draw two arcs either side of the point of equal width (giving points S and T) 3. Place the compass on point S, open over halfway and draw an arc above the line. 4. Repeat from the other arc on the line (point T). 5. Draw a straight line from the intersecting arcs to the original point on the line.	
8. Constructing Triangles (Side, Side, Side)	1. Draw the base of the triangle using a ruler. 2. Open a pair of compasses to the width of one side of the triangle. 3. Place the point on one end of the line and draw an arc. 4. Repeat for the other side of the triangle at the other end of the line. 5. Using a ruler, draw lines connecting the ends of the base of the triangle to the point where the arcs intersect.	
9. Constructing Triangles (Side, Angle, Side)	1. Draw the base of the triangle using a ruler. 2. Measure the angle required using a protractor and mark this angle. 3. Remove the protractor and draw a line of the exact length required in line with the angle mark drawn. 4. Connect the end of this line to the other end of the base of the triangle.	
10. Constructing Triangles (Angle, Side, Angle)	1. Draw the base of the triangle using a ruler. 2. Measure one of the angles required using a protractor and mark this angle. 3. Draw a straight line through this point from the same point on the base of the triangle. 4. Repeat this for the other angle on the other end of the base of the triangle.	

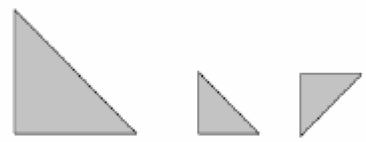
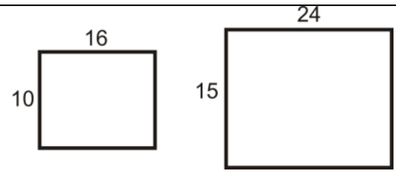
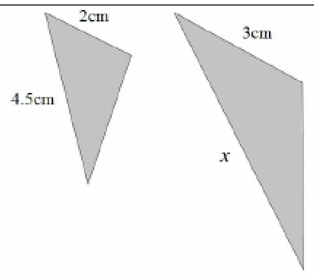
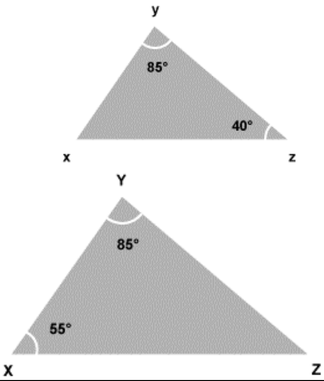
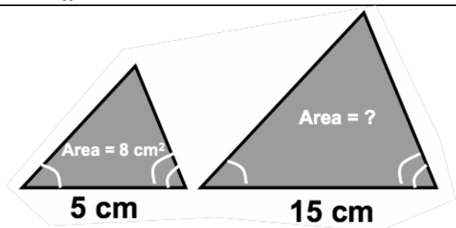
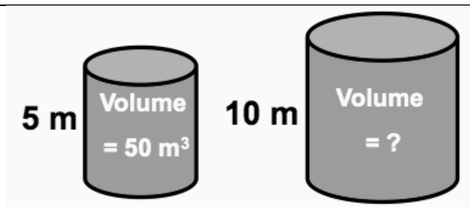
18. Construction & Congruence

11. Constructing an Equilateral Triangle (also makes a 60° angle)	1. Draw the base of the triangle using a ruler. 2. Open the pair of compasses to the exact length of the side of the triangle. 3. Place the sharp point on one end of the line and draw an arc. 4. Repeat this from the other end of the line. 5. Using a ruler, draw lines connecting the ends of the base of the triangle to the point where the arcs intersect.	
12. Loci and Regions	A **locus** is a **path of points that follow a rule**. For the locus of points **closer to B than A**, create a **perpendicular bisector** between A and B and shade the side closer to B. For the locus of points **equidistant from A**, use a compass to draw a **circle**, centre A. For the locus of points **equidistant to line X and line Y**, create an **angle bisector**. For the locus of points a set **distance from a line**, create **two semi-circles** at either end joined by **two parallel lines**.	 Points Closer to B than A  Points less than 2cm from A Points more than 2cm from A  
13. Equidistant	A point is equidistant from a set of objects if the **distances between that point and each of the objects is the same**.	

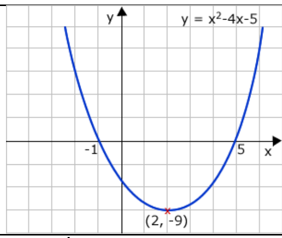
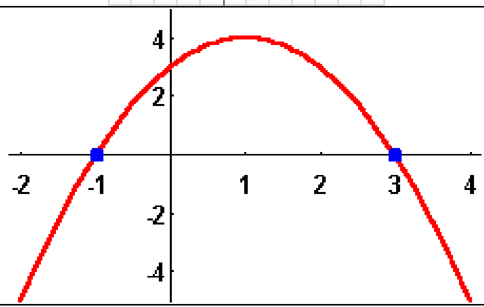
18. Construction & Congruence

14. Congruent Shapes	Shapes are congruent if they are **identical** - **same shape** and **same size**. Shapes can be rotated or reflected but still be congruent.	
15. Congruent Triangles	4 ways of proving that two triangles are congruent: 1. **SSS** (Side, Side, Side) 2. **RHS** (Right angle, Hypotenuse, Side) 3. **SAS** (Side, Angle, Side) 4. **ASA** (Angle, Side, Angle) or **AAS** <u>(AAA proves similarity, not congruency)</u>	 $BC = DF$ $\angle ABC = \angle EDF$ $\angle ACB = \angle EFD$ $\therefore$ The two triangles are congruent by AAS.
16. Similar Shapes	Shapes are similar if they are the **same shape but different sizes**. The proportion of the matching sides must be the same, meaning the ratios of corresponding sides are all equal.	

19. Similar shapes

Topic/Skill	Definition/Tips	Example
1. Similar Shapes	Shapes are similar if they are the **same shape but different sizes**. The proportion of the matching sides must be the same, meaning the ratios of corresponding sides are all equal.	
2. Scale Factor	The **ratio of corresponding sides** of two similar shapes. To find a scale factor, **divide a length** on one shape **by the corresponding length** on a similar shape (big/small).	 Scale Factor = $15 \div 10 = 1.5$
3. Finding missing lengths in similar shapes	1. Find the **scale factor**. 2. **Multiply or divide** the corresponding side to find a missing length. If you are finding a missing length on the **larger** shape - **multiply** by the scale factor. If you are finding a missing length on the **smaller** shape - **divide** by the scale factor.	 Scale Factor = $3 \div 2 = 1.5$ $x = 4.5 \times 1.5 = 6.75cm$
4. Similar Triangles	To show that two triangles are **similar**, show that: 1. The **three sides** are in the **same proportion** 2. **Two sides** are in the **same proportion**, and their **included angle** is the **same** 3. The **three angles** are **equal**	
5. Scale Factor for Area	If linear scale factor is **a**, **area** scale factor is $\mathbf{a^2}$	 sf(linear) = 15/3= 5, sf^2 = 25 area = 8x25 = 200cm^2
6. Scale Factor for Volume	If linear scale factor is **a**, **volume** scale factor is $\mathbf{a^3}$	 sf(linear) = 10/5 = 2, sf^3 = 23 = 8 volume = 50x8 = 400m^3

20. Quadratic Equations

Topic/Skill	Definition/Tips	Example
1. Quadratic	A quadratic expression is of the form $\boldsymbol{ax^2+bx+c}$ where a, b and c are numbers, $\boldsymbol{a \neq 0}$	Examples of quadratic expressions: x^2 $8x^2-3x+7$ Examples of non-quadratic expressions: $2x^3-5x^2$ $9x-1$
2. Factorising Quadratics	When a quadratic expression is in the form x^2+bx+c find the two numbers that **add to give b** and **multiply to give c**.	$x^2+7x+10=(x+5)(x+2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2+2x-8=(x+4)(x-2)$ (because +4 and -2 add to give +2 and multiply to give -8)
3. Difference of Two Squares	An expression of the form $\boldsymbol{a^2-b^2}$ can be factorised to give $(\boldsymbol{a+b})(\boldsymbol{a-b})$	$x^2-25=(x+5)(x-5)$ $16x^2-81=(4x+9)(4x-9)$
4. Solving Quadratics ($\boldsymbol{ax^2=b}$)	Isolate the x^2 term and square root both sides. Remember there will be a **positive and a negative solution**.	$2x^2=98$ $x^2=49$ $x=\pm 7$
5. Solving Quadratics ($\boldsymbol{ax^2+bx=0}$)	**Factorise** and then **solve = 0**.	$x^2-3x=0$ $x(x-3)=0$ $x=0 \text{ or } x=3$
6. Solving Quadratics by Factorising ($\boldsymbol{a=1}$)	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $x^2+3x-10=0$ Factorise: $(x+5)(x-2)=0$ $x=-5 \text{ or } x=2$
7. Quadratic Graph	A '**U-shaped**' curve called a **parabola**. The equation is of the form $y=\boldsymbol{ax^2+bx+c}$, where a, b and c are numbers, $\boldsymbol{a \neq 0}$. If $\boldsymbol{a<0}$, the parabola is **upside down**.	
8. Roots of a Quadratic	A root is a **solution**. The roots of a quadratic are the ***x*-intercepts of the quadratic graph**.	

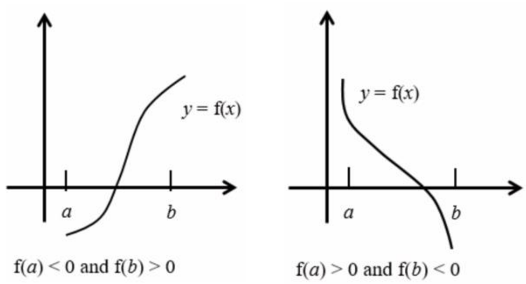
20. Quadratic Equations

9. Turning Point of a Quadratic	A turning point is the **point where a quadratic turns**. On a **positive parabola**, the turning point is called a **minimum**. On a **negative parabola**, the turning point is called a **maximum**.	
10. Factorising Quadratics when $a \neq 1$	When a quadratic is in the form $ax^2 + bx + c$ 1. Multiply a by c = ac 2. Find two numbers that add to give b and multiply to give ac. 3. Re-write the quadratic, replacing bx with the two numbers you found. 4. Factorise in pairs – you should get the same bracket twice 5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.	Factorise $6x^2 + 5x - 4$ 1. $6 \times -4 = -24$ 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 3. $6x^2 + 8x - 3x - 4$ 4. Factorise in pairs: $2x(3x + 4) - 1(3x + 4)$ 5. Answer = $(3x + 4)(2x - 1)$
11. Solving Quadratics by Factorising ($a \neq 1$)	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $2x^2 + 7x - 4 = 0$ Factorise: $(2x - 1)(x + 4) = 0$ $x = \frac{1}{2}$ or $x = -4$
12. Completing the Square (when $a = 1$)	A quadratic in the form $x^2 + bx + c$ can be written in the form $(\boldsymbol{x} + \boldsymbol{p})^2 + \boldsymbol{q}$ 1. Write a set of brackets with x in and **half** the value of b. 2. Square the bracket. 3. Subtract $\left(\frac{b}{2}\right)^2$ and add c. 4. Simplify the expression. You can **use the completing the square form** to help **find the maximum or minimum** of quadratic graph.	Complete the square of $y = x^2 - 6x + 2$ Answer: $(x - 3)^2 - 3^2 + 2$ $= (x - 3)^2 - 7$ The minimum value of this expression occurs when $(x - 3)^2 = 0$, which occurs when $x = 3$ When $x = 3$, $y = 0 - 7 = -7$ Minimum point = $(3, -7)$
13. Completing the Square (when $a \neq 1$)	A quadratic in the form $ax^2 + bx + c$ can be written in the form $\mathbf{p}(\boldsymbol{x} + \boldsymbol{q})^2 + \boldsymbol{r}$ Use the same method as above, but factorise out a at the start.	Complete the square of $4x^2 + 8x - 3$ Answer: $4[x^2 + 2x] - 3$ $= 4[(x + 1)^2 - 1^2] - 3$ $= 4(x + 1)^2 - 4 - 3$ $= 4(x + 1)^2 - 7$

20. Quadratic Equations

14. Algebraic Fraction	A fraction whose **numerator** and **denominator** are **algebraic expressions**.	$\frac{6x}{3x-1}$
15. Adding/ Subtracting Algebraic Fractions	For $\frac{a}{b} \pm \frac{c}{d}$, the **common denominator** is bd $\frac{a}{b} \pm \frac{c}{d} = \frac{ad}{bd} \pm \frac{bc}{bd} = \frac{ad \pm bc}{bd}$	$\frac{1}{x} + \frac{x}{2y}$ $= \frac{1(2y)}{2xy} + \frac{x(x)}{2xy}$ $= \frac{2y + x^2}{2xy}$
16. Multiplying Algebraic Fractions	**Multiply** the **numerators together** and the **denominators together**. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$	$\frac{x}{3} \times \frac{x+2}{x-2}$ $= \frac{x(x+2)}{3(x-2)}$ $= \frac{x^2+2x}{3x-6}$
17. Dividing Algebraic Fractions	**Multiply** the first fraction by the **reciprocal of the second fraction**. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$	$\frac{x}{3} \div \frac{2x}{7}$ $= \frac{x}{3} \times \frac{7}{2x}$ $= \frac{7x}{6x} = \frac{7}{6}$
18. Simplifying Algebraic Fractions	**Factorise** the numerator and denominator and **cancel common factors**.	$\frac{x^2+x-6}{2x-4} = \frac{(x+3)(x-2)}{2(x-2)} = \frac{x+3}{2}$
19. Coefficient	A **number** used to **multiply** a **variable**. It is the number that comes before/in front of a letter.	6z 6 is the coefficient z is the variable
20. Odds and Evens	An **even** number is a **multiple of 2** An **odd** number is an integer which is **not a multiple of 2**.	If n is an integer (whole number): An even number can be represented by **2n** or **2m** etc. An odd number can be represented by **2n-1** or **2n+1** or **2m+1** etc.
21. Consecutive Integers	Whole numbers that follow each other in order.	If n is an integer: **n, n+1, n+2** etc. are consecutive integers.
22. Square Terms	A term that is produced by multiply another term by itself.	If n is an integer: n^2, m^2 etc. are square integers

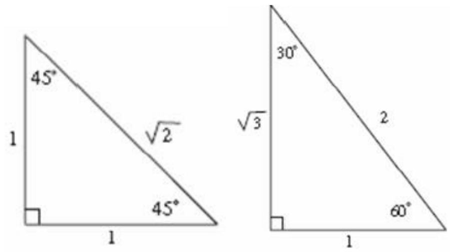
20. Quadratic Equations

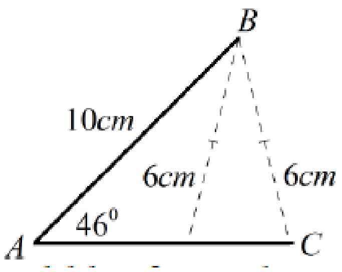
23. Sum	The sum of two or more numbers (or variables) is the value you get when you add them together.	The sum of 4 and 6 is 10 n + n = 2n
24. Product	The product of two or more numbers (or variables) is the value you get when you multiply them together.	The product of 4 and 6 is 24 n x m = nm
25. Multiple	To show that an expression is a **multiple** of a number, you need to show that you can **factor out that number**.	$4n^2 + 8n - 12$ is a multiple of 4 because it can be written as: $4(n^2 + 2n - 3)$
26. Iteration	The act of **repeating a process** over and over again, often with the aim of **approximating** a desired result more closely. **Recursive** Notation: $x_{n+1} = \sqrt{3x_n + 6}$	$x_1 = 4$ $x_2 = \sqrt{3 \times 4 + 6} = 4.242640 \ldots$ $x_3 = \sqrt{3 \times 4.242640 \ldots + 6}$ $= 4.357576 \ldots$
27. Iterative Method	To create an iterative formula, **rearrange** an equation with more than one x term to **make one of the x terms the subject**. You will be given the first value to substitute in, often called $\boldsymbol{x_1}$. **Keep substituting in your previous answer** until your answers are the same to a certain degree of accuracy. This is called converging to a limit. Use the 'ANS' button on your calculator to keep substituting in the previous answer.	Use an iterative formula to find the positive root of $x^2 - 3x - 6 = 0$ to 3 decimal places. $x_1 = 4$ Answer: $x^2 = 3x + 6$ $x = \sqrt{3x + 6}$ So $x_{n+1} = \sqrt{3x_n + 6}$ $x_1 = 4$ $x_2 = \sqrt{3 \times 4 + 6} = 4.242640 \ldots$ $x_3 = \sqrt{3 \times 4.242640 \ldots + 6}$ $= 4.357576 \ldots$ Keep repeating… $x_7 = 4.372068.. = 4.372\ (3dp)$ $x_8 = 4.372208 \ldots = 4.372\ (3dp)$ So answer is $x = 4.372\ (3dp)$
28. Solution/root	For a function, a **solution** or **root** is where the graph crosses the **x-axis**. To show a solution occurs between any two consecutive numbers (a & b), the graph will be positive at one of them, then negative at the next (or vice versa). **'There has been a change of sign, therefore there is a solution between a & b.'**	

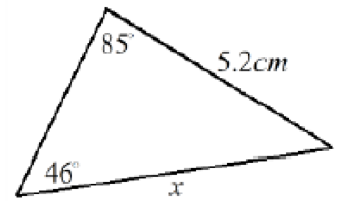
20. Quadratic Equations

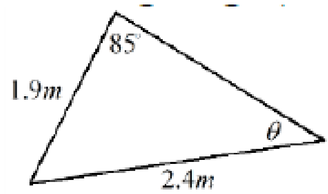
29. Show a rearrangement	Show an equation can be **re-arranged** into another format (which will then be used for the iterative process.) You must show **each** intermediate step clearly.	Show that $x^3 + 5x - 1 = 0$ can be arranged to give $x = \frac{1-x^3}{5}$ $5x = 1 - x^3$ $x = \dfrac{1-x^3}{5}$
30. Completing the Square (when $\boldsymbol{a = 1}$)	A quadratic in the form $x^2 + bx + c$ can be written in the form $\boldsymbol{(x+p)^2 + q}$ 1. Write a set of brackets with x in and **half** the value of b. 2. Square the bracket. 3. Subtract $\left(\frac{b}{2}\right)^2$ and add c. 4. Simplify the expression. You can **use the completing the square form** to help **find the maximum or minimum** of quadratic graph.	Complete the square of $y = x^2 - 6x + 2$ Answer: $(x-3)^2 - 3^2 + 2$ $= (x-3)^2 - 7$ The minimum value of this expression occurs when $(x-3)^2 = 0$, which occurs when $x = 3$ When $x = 3$, $y = 0 - 7 = -7$ Minimum point = $(3, -7)$
31. Completing the Square (when $\boldsymbol{a \neq 1}$)	A quadratic in the form $ax^2 + bx + c$ can be written in the form $\mathbf{p}\boldsymbol{(x+q)^2 + r}$ Use the same method as above, but factorise out a at the start. Note, you only need to factorise the a out of the first two terms.	Complete the square of $4x^2 + 8x - 3$ Answer: $4[x^2 + 2x] - 3$ $= 4[(x+1)^2 - 1^2] - 3$ $= 4(x+1)^2 - 4 - 3$ $= 4(x+1)^2 - 7$
32. Solving Quadratics by Completing the Square	**Complete the square** in the usual way and **use inverse operations to solve**.	Solve $x^2 + 8x + 1 = 0$ Answer: $(x+4)^2 - 4^2 + 1 = 0$ $(x+4)^2 - 15 = 0$ $(x+4)^2 = 15$ $(x+4) = \pm\sqrt{15}$ $x = -4 \pm \sqrt{15}$
33. Solving Quadratics using the Quadratic Formula	A quadratic in the form $ax^2 + bx + c = 0$ can be solved using the formula: $\boldsymbol{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ Use the formula if the quadratic does not factorise easily. Use it when a rounding instruction is given in the question.	Solve $3x^2 + x - 5 = 0$ Answer: $a = 3, b = 1, c = -5$ $x = \dfrac{-1 \pm \sqrt{1^2 - 4 \times 3 \times -5}}{2 \times 3}$ $x = \dfrac{-1 \pm \sqrt{61}}{6}$ $x = 1.14 \; or -1.47 \; (2 \; d.p.)$

21. Further Trigonometry

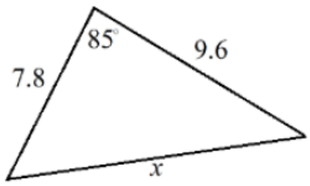
<table>
<tr><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th></tr>
<tr><td>1. Exact Values for Angles in Trigonometry</td><td>
<table>
<tr><th></th><th>0°</th><th>30°</th><th>45°</th><th>60°</th><th>90°</th></tr>
<tr><td>sin</td><td>0</td><td>$\frac{1}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{\sqrt{3}}{2}$</td><td>1</td></tr>
<tr><td>cos</td><td>1</td><td>$\frac{\sqrt{3}}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{1}{2}$</td><td>0</td></tr>
<tr><td>tan</td><td>0</td><td>$\frac{1}{\sqrt{3}}$</td><td>1</td><td>$\sqrt{3}$</td><td>----</td></tr>
</table>
</td><td>
</td></tr>
<tr><td>2. Sine Rule</td><td>Use with non right angle triangles.
Use when the question involves 2 sides and 2 angles.

For missing side:
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
For missing angle:
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
There is an ambiguous case (where there are two potential answers)


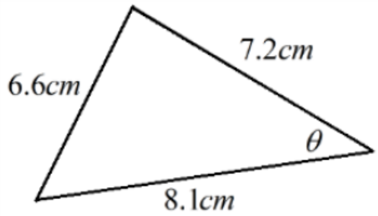
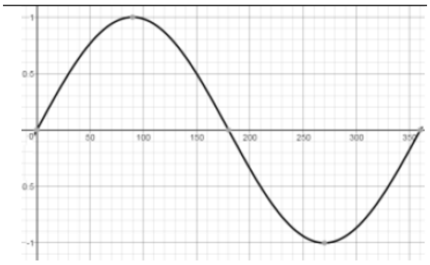
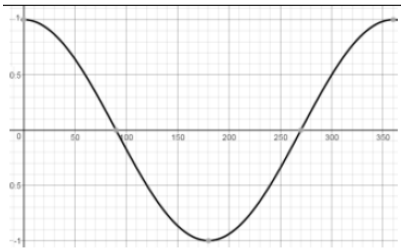
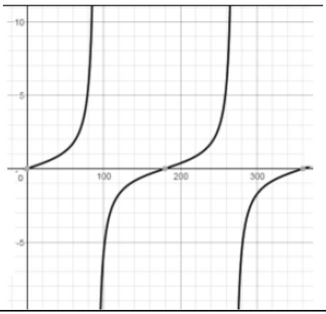
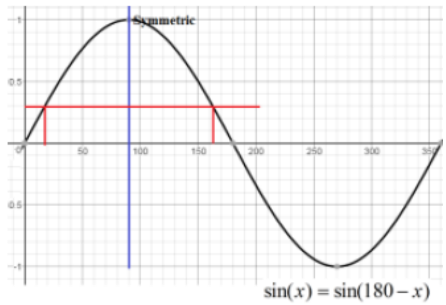
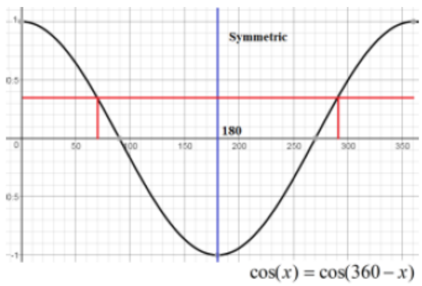
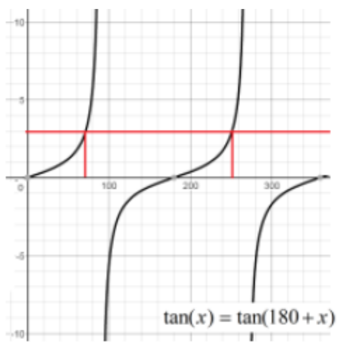
To find the two angles, use sine to find one, and then subtract your answer from 180 to find the other answer.</td><td>

$$\frac{x}{\sin 85} = \frac{5.2}{\sin 46}$$
$$x = \frac{5.2 \times \sin 85}{\sin 46} = 3.75cm$$


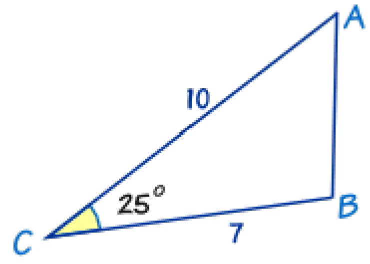
$$\frac{\sin\theta}{1.9} = \frac{\sin 85}{2.4}$$
$$\sin\theta = \frac{1.9 \times \sin 85}{2.4} = 0.789$$
$$\theta = sin^{-1}(0.789) = 52.1°$$</td></tr>
<tr><td>3. Cosine Rule</td><td>Use with non right angle triangles.
Use when the question involves 3 sides and 1 angle.

For missing side:
$$a^2 = b^2 + c^2 - 2bccosA$$
For missing angle:
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$</td><td>

$$x^2 = 9.6^2 + 7.8^2 - (2 \times 9.6 \times 7.8 \times \cos 85)$$
$$x = 11.8$$</td></tr>
</table>

		 $\cos\theta = \dfrac{7.2^2 + 8.1^2 - 6.6^2}{2 \times 7.2 \times 8.1}$ $\theta = 50.7°$
4. Graphs of Trigonometric Functions	$y = \sin(x)$ for $0 \le x \le 360^o$  $y = \cos(x)$ for $0 \le x \le 360^o$  $y = \tan(x)$ for $0 \le x \le 360^o$ 	  

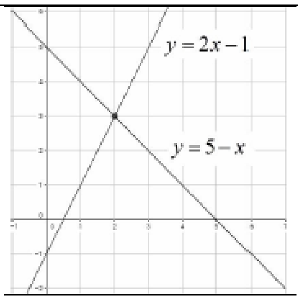
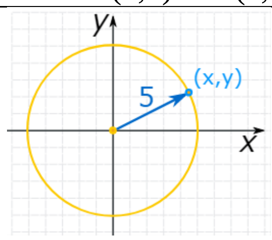
21. Further Trigonometry

5. Area of a Triangle	Use when given the **length of two sides and the included angle**. $$\boldsymbol{Area\ of\ a\ Triangle = \frac{1}{2}ab\sin C}$$	 $A = \frac{1}{2}ab\sin C$ $A = \frac{1}{2} \times 7 \times 10 \times \sin 25$ $A = 14.8$

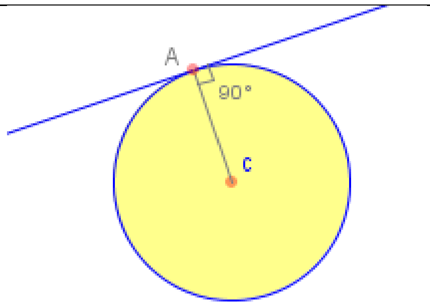
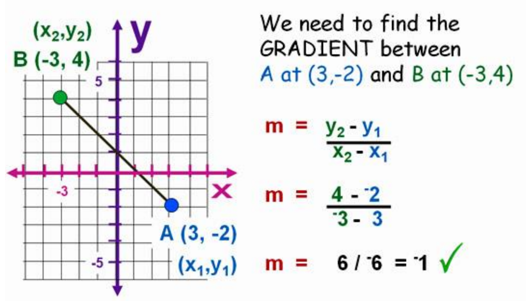
23. Simultaneous Equations

Topic/Skill	Definition/Tips	Example
1. Simultaneous Equations	A set of **two or more equations**, each involving **two or more variables** (letters). The **solutions** to simultaneous equations **satisfy both**/all of the **equations**.	$2x + y = 7$ $3x - y = 8$ $x = 3$ $y = 1$
2. Variable	A **symbol**, usually a **letter**, which **represents a number** which is usually unknown.	In the equation $x + 2 = 5$, x is the variable.
3. Coefficient	A **number** used to **multiply** a **variable**. It is the number that comes before/in front of a letter.	6z 6 is the coefficient, z is the variable
4. Solving Simultaneous Equations (by Elimination)	1. **Balance** the **coefficients** of one of the variables (the middle variable is the safest one to use). 2. **Eliminate** this variable by adding or subtracting the equations (**Add If Different Signs, Minus If Same Sign**) 3. **Solve** the linear equation you get using the other variable. 4. **Substitute** the value you found back into one of the previous equations. 5. **Solve** the equation you get. 6. **Check** that the two values you get satisfy both of the original equations.	$5x + 2y = 9$ $10x + 3y = 16$ Multiply the first equation by 2. $10x + 4y = 18$ $10x + 3y = 16$ Same Sign Subtract (+10x on both) $y = 2$ Substitute $y = 2$ in to equation. $5x + 2 \times 2 = 9$ $5x + 4 = 9$ $5x = 5$ $x = 1$ Solution: $x = 1, y = 2$
5. Solving Simultaneous Equations (by Substitution)	1. **Rearrange** one of the equations into the form $y = \ldots$ or $x = \ldots$ 2. **Substitute** the right-hand side of the rearranged equation into the other equation. 3. Expand and **solve** this equation. 4. **Substitute** the value into the $y = \ldots$ or $x = \ldots$ equation. 5. **Check** that the two values you get satisfy both of the original equations.	$y - 2x = 3$ $3x + 4y = 1$ Rearrange: $y - 2x = 3 \rightarrow y = 2x + 3$ Substitute: $3x + 4(2x + 3) = 1$ Solve: $3x + 8x + 12 = 1$ $11x = -11$ $x = -1$ Substitute: $y = 2 \times -1 + 3$ $y = 1$ Solution: $x = -1, y = 1$

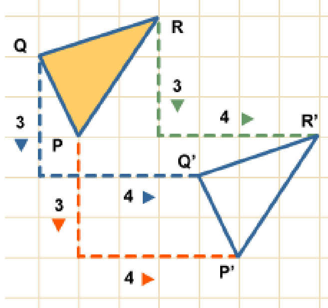
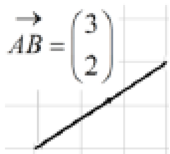
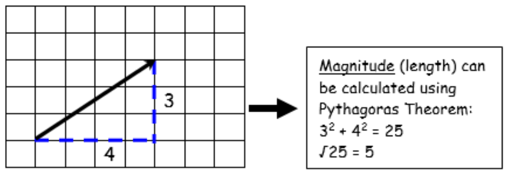
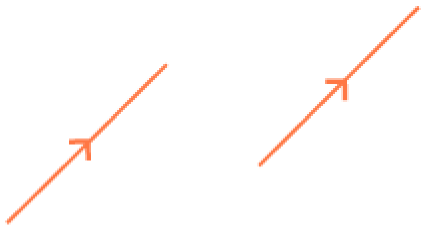
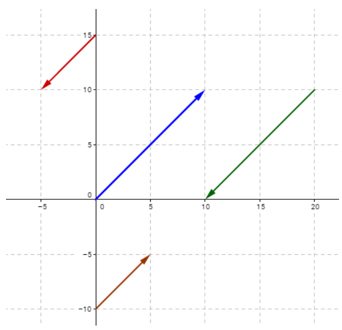
23. Simultaneous Equations

6. Solving Simultaneous Equations (Graphically)	**Draw the graphs** of the two equations. The **solutions** will be **where the lines meet**. The solution can be written as a **coordinate**.	 $y = 5 - x$ and $y = 2x - 1$. They meet at the point with coordinates (2,3) so the answer is $x = 2$ and $y = 3$
7. Solving Linear and Quadratic Simultaneous Equations	Method 1: If both equations are in the same form (eg. Both $y = \ldots$): 1. Set the equations **equal to each other**. 2. **Rearrange** to make the equation **equal to zero**. 3. **Solve** the quadratic equation. 4. **Substitute** the values back in to one of the equations. Method 2: If the equations are not in the same form: 1. **Rearrange** the linear equation into the form $y = \ldots$ or $x = \ldots$ 2. **Substitute** in to the quadratic equation. 3. **Rearrange** to make the equation **equal to zero**. 4. **Solve** the quadratic equation. 5. **Substitute** the values back in to one of the equations. You should get **two pairs of solutions** (two values for x, two values for y.) Graphically, you should have **two points of intersection**.	Example 1 Solve $y = x^2 - 2x - 5$ and $y = x - 1$ $x^2 - 2x - 5 = x - 1$ $x^2 - 3x - 4 = 0$ $(x-4)(x+1) = 0$ $x = 4$ and $x = -1$ $y = 4 - 1 = 3$ and $y = -1 - 1 = -2$ Answers: (4,3) and (-1,-2) Example 2 Solve $x^2 + y^2 = 5$ and $x + y = 3$ $x = 3 - y$ $(3 - y)^2 + y^2 = 5$ $9 - 6y + y^2 + y^2 = 5$ $2y^2 - 6y + 4 = 0$ $y^2 - 3y + 2 = 0$ $(y-1)(y-2) = 0$ $y = 1$ and $y = 2$ $x = 3 - 1 = 2$ and $x = 3 - 2 = 1$ Answers: (2,1) and (1,2)
8. Equation of a Circle	The equation of a **circle**, **centre (0,0), radius r**, is: $x^2 + y^2 = r^2$	 $x^2 + y^2 = 25$

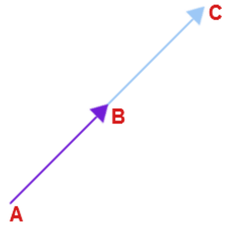
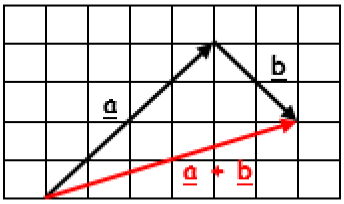
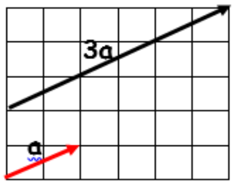
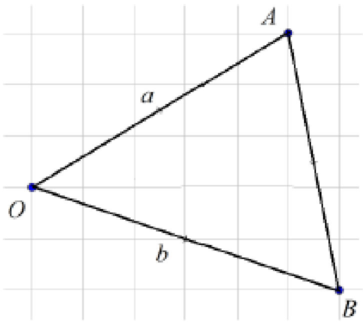
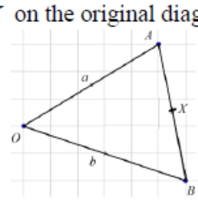
23. Simultaneous Equations

9. Tangent	A straight **line** that **touches** a circle at **exactly one point**, never entering the circle's interior. A **radius** is **perpendicular** to a **tangent** at the **point of contact** (see circle theorems).	
10. Gradient	**Gradient** is another word for **slope**. $G = \frac{Rise}{Run} = \frac{Change\ in\ y}{Change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$	

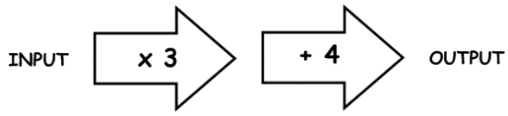
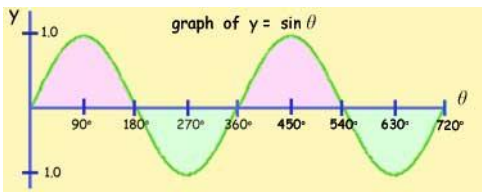
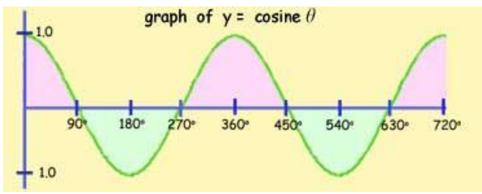
24. Vector geometry

Topic/Skill	Definition/Tips	Example
1. Translation	**Translate** means to **move a shape**. The shape does not change **size** or **orientation**.	
2. Vector Notation	A vector can be written in 3 ways: **a** or $\overrightarrow{\boldsymbol{AB}}$ or $\begin{pmatrix}\mathbf{1}\\\mathbf{3}\end{pmatrix}$	
3. Column Vector	In a column vector, the **top** number moves **left (-) or right (+)** and the **bottom** number moves **up (+) or down (-)**	$\begin{pmatrix}2\\3\end{pmatrix}$ means '2 right, 3 up' $\begin{pmatrix}-1\\-5\end{pmatrix}$ means '1 left, 5 down'
4. Vector	A **vector** is a quantity represented by an arrow with both **direction** and **magnitude**. $\overrightarrow{AB} = -\overrightarrow{BA}$	
5. Magnitude	Magnitude is defined as the **length** of a vector.	
6. Equal Vectors	If two vectors have the **same magnitude and direction**, they are **equal**.	
7. Parallel Vectors	**Parallel** vectors are **multiples** of each other.	2**a**+**b** and 4**a**+2**b** are parallel as they are multiple of each other. 

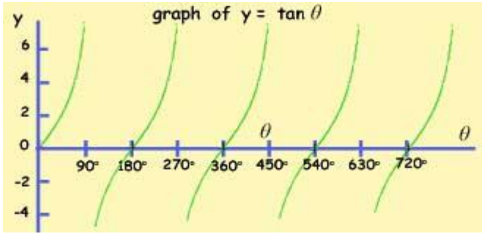
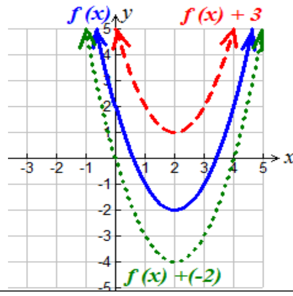
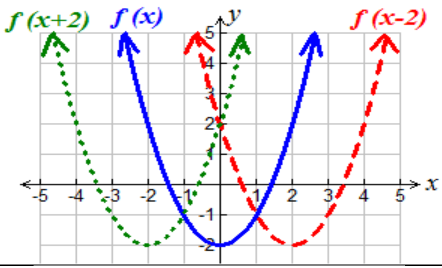
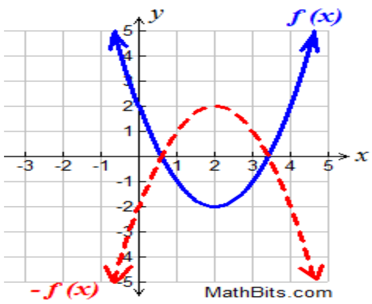
24. Vector geometry

8. Collinear Vectors	**Collinear** vectors are vectors that are on the **same line**. To show that two vectors are **collinear**, show that one vector is a **multiple** of the other (parallel) **AND** that both vectors **share a point**.	
9. Resultant Vector	The **resultant** vector is the vector that results from **adding** two or more vectors together. The resultant can also be shown by **lining up** the **head** of one vector with the **tail** of the other.	if $\underline{a} = \begin{pmatrix}4\\4\end{pmatrix}$ and $\underline{b} = \begin{pmatrix}2\\-2\end{pmatrix}$ then $\underline{a} + \underline{b} = \begin{pmatrix}4\\4\end{pmatrix} + \begin{pmatrix}2\\-2\end{pmatrix} = \begin{pmatrix}6\\2\end{pmatrix}$ 
10. Scalar of a Vector	A **scalar** is the **number** we **multiply** a vector by.	 Example: 3a + 2b = $= 3\begin{pmatrix}2\\1\end{pmatrix} + 2\begin{pmatrix}4\\-1\end{pmatrix}$ $= \begin{pmatrix}6\\3\end{pmatrix} + \begin{pmatrix}8\\-2\end{pmatrix}$ $= \begin{pmatrix}14\\1\end{pmatrix}$
11. Vector Geometry	 $\overrightarrow{OA} = a \quad \overrightarrow{AO} = -a$ $\overrightarrow{OB} = b \quad \overrightarrow{BO} = -b$ $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b = b - a$ $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -b + a = a - b$	**Example 1:** X is the midpoint of AB. Find $\overrightarrow{OX}$ **Answer:** Draw X on the original diagram  Now build up a journey. You could use $\overrightarrow{OX} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$. This will give: $\overrightarrow{OX} = a + \frac{1}{2}(b-a)$. This will simplify to $\frac{1}{2}a + \frac{1}{2}b$ or $\frac{1}{2}(a+b)$

25. Transform Functions

Topic/Skill	Definition/Tips	Example
1. Function Machine	Takes an **input** value, performs some **operations** and produces an **output** value.	
2. Function	A **relationship** between two sets of values.	$f(x) = 3x^2 - 5$ 'For any input value, square the term, then multiply by 3, then subtract 5'.
3. Function notation	$f(x)$ $\boldsymbol{x}$ is the **input** value $\boldsymbol{f(x)}$ is the **output** value.	$f(x) = 3x + 11$ Suppose the input value is $x = 5$ The output value is $f(5) = 3 \times 5 + 11 = 26$
4. Inverse function	$f^{-1}(x)$ A function that performs the **opposite process** of the original function. 1. Write the function as $y = f(x)$ 2. Rearrange to make x the subject. 3. Replace the $\boldsymbol{y}$ **with** $\boldsymbol{x}$ and the $\boldsymbol{x}$ **with** $\boldsymbol{f^{-1}(x)}$	$f(x) = (1 - 2x)^5$. Find the inverse. $y = (1 - 2x)^5$ $\sqrt[5]{y} = 1 - 2x$ $1 - \sqrt[5]{y} = 2x$ $\frac{1 - \sqrt[5]{y}}{2} = x$ $f^{-1}(x) = \frac{1 - \sqrt[5]{x}}{2}$
5. Composite function	A **combination** of two or more **functions** to create a new function. $\boldsymbol{fg(x)}$ is the composite function that **substitutes** the function $\boldsymbol{g(x)}$ **into** the function $\boldsymbol{f(x)}$. $\boldsymbol{fg(x)}$ means '**do g first, then f**' $\boldsymbol{gf(x)}$ means '**do f first, then g**'	$f(x) = 5x - 3,\ g(x) = \frac{1}{2}x + 1$ What is $fg(4)$? $g(4) = \frac{1}{2} \times 4 + 1 = 3$ $f(3) = 5 \times 3 - 3 = 12 = fg(4)$ What is $fg(x)$? $fg(x) = 5\left(\frac{1}{2}x + 1\right) - 3 = \frac{5}{2}x + 2$
6. $\boldsymbol{y = \sin x}$	Key Coordinates: $\boldsymbol{(0,0), (90,1), (180,0), (270,-1), (360,0}$ y is never more than 1 or less than -1. Pattern repeats every 360°.	
7. $\boldsymbol{y = \cos x}$	Key Coordinates: $\boldsymbol{(0,1), (90,0), (180,-1), (270,0), (360,1}$ y is never more than 1 or less than -1. Pattern repeats every 360°.	

25. Transform Functions

8. $y = \tan x$	Key Coordinates: $(0, 0), (45, 1), (135, -1), (180, 0), (225, 1), (315, -1), (360, 0)$ **Asymptotes** at $x = 90$ and $x = 270$ Pattern repeats every 360°.	
9. $f(x) + a$	**Vertical translation** up a units. $\begin{pmatrix} 0 \\ a \end{pmatrix}$	
10. $f(x + a)$	**Horizontal translation** <u>left</u> a units. $\begin{pmatrix} -a \\ 0 \end{pmatrix}$	
11. $-f(x)$	**Reflection** over the **x-axis**.	
12. $f(-x)$	**Reflection** over the **y-axis**.